PHYSICS

GRADE 11

STUDENT TEXTBOOK

CONTENTS

PHYSICS11

SEMESTER-1 (Period-I)

P11CH1

TOPIC

1

Motion in Two Dimensions

1.1. SCALAR AND VECTOR QUANTITIES

All those quantities which can be measured are known as ***physical quantities***. These quantities can be broadly classified into two categories—scalar quantities and vector quantities.

Scalar quantities *are those physical quantities which have only magnitude and no direction.*

These obey the ordinary laws of Algebra. A scalar quantity is completely specified by merely stating a number. A few *examples* of scalars are volume, mass, speed, density, temperature, pressure, time, power, total path length and energy.

Vector quantities *are those physical quantities which have both magnitude and direction and obey the laws of vector addition.*

A vector is specified not by merely stating a number but a direction as well. Since the concept of vectors involves the idea of direction, therefore, vectors do not follow the ordinary laws of Algebra. A few examples of vectors are displacement, velocity, acceleration, impulse, force and linear momentum.

1.2. REPRESENTATION OF A VECTOR

A vector is represented by a line with an arrow head. In Fig. 1.1, a vector $\vec{a}$ is represented by a directed line PQ. The length of the line gives the magnitude of the vector. The magnitude of the vector is called the modulus of the vector. The direction of the arrow represents the direction of the vector.

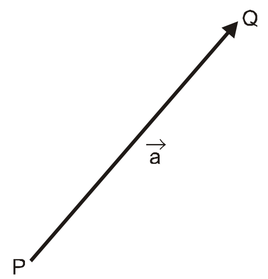


Fig. 1.1. Representation of a vector

1.3. IMPORTANT TERMS

(*i*) ***Parallel vectors.*** If two collinear vectors $\vec{a}$ and $\vec{b}$ act in the same direction, then the angle between them is 0°. When vectors act along the same direction, they are called parallel vectors.

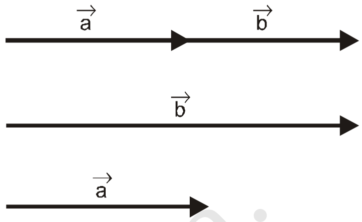


Fig. 1.2. Parallel vectors

(*ii*) ***Antiparallel vectors.*** If two collinear vectors act in opposite directions, then the angle between them is 180° or π radian. Vectors are said to be anti-parallel if they act in opposite directions.

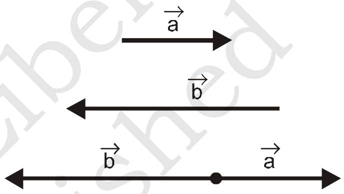


Fig. 1.3. Antiparallel vectors

(*iii*) ***Unit vector*** *of a given vector is a vector of unit magnitude and has the same direction as that of the given vector.*

Unit vector is used to denote the direction of a given vector. It is unitless and dimensionless vector.

Any vector $\vec{a}$ can be expressed in terms of its unit vector $\hat{a}$ as follows:

$$\vec{a} = a\,\hat{a}$$

Here $\hat{a}$ is in the direction of $\vec{a}$. $\hat{a}$ is read as '*a* hat' or '*a* cap'.

$$\hat{a} = \frac{\vec{a}}{a} \quad \text{or} \quad \frac{\vec{a}}{|\vec{a}|}$$

So, if a given vector is divided by its magnitude, we get a unit vector.

(*iv*) The ***three rectangular unit vectors*** $\hat{i}$, $\hat{j}$ and $\hat{k}$ are shown in Fig. 1.4. $\hat{i}$ denotes the direction of X-axis. $\hat{j}$ denotes the direction of Y-axis and $\hat{k}$ denotes the direction of Z-axis. The three unit vectors $\hat{i}$,

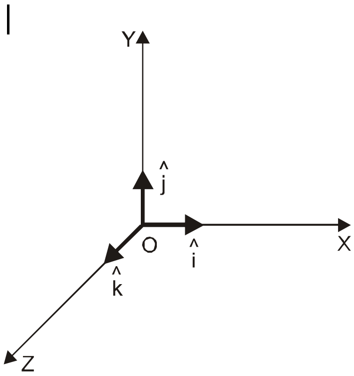


Fig. 1.4. Orthogonal triad of unit vectors

$\hat{j}$ and $\hat{k}$ are collectively known as '*orthogonal triad of unit vectors*'. These are also known as *base vectors.*

(*v*) ***Negative of a vector.*** *A vector is said to be* **negative** *of a given vector if its magnitude is the same as that of the given vector but direction is reversed.*

The negative of a vector $\vec{a}$ is denoted by '$-\vec{a}$'.

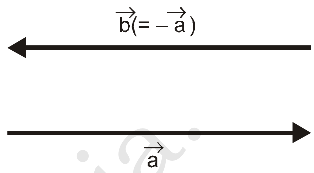


Fig. 1.5. Negative vector

In Fig. 1.5, $\vec{b}$ is the negative of $\vec{a}$. $\vec{b} = -\vec{a}$

1.4. POSITION VECTOR

A vector which gives the position of a point with reference to the origin of the co-ordinate system is called ***position vector.***

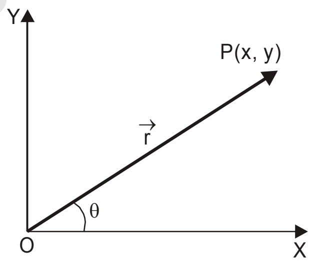


Fig. 1.6. Position vector in two dimensions

Consider a particle moving in a plane. To describe the position of this particle at any time *t*, we use a vector called *position vector.* This *helps to locate the position* of a particle moving in plane or even in space. Suppose at any instant of time, the particle is at P. Then $\vec{OP}$ is the position vector which gives the position of the particle with reference to a point O in the plane of motion. This point O has been chosen as the origin.

The magnitude of the position vector gives the distance of the particle from some arbitrarily chosen origin. In addition to this, the direction of the position vector gives us the direction θ in which P lies as viewed from O.

It may be noted here that position vectors will be different for different positions of the particle.

The position vector $\vec{r}$ at any time *t*, in terms of co-ordinates *x* and *y*, is given by,

$$\vec{r} = \vec{x} + \vec{y} \quad \text{or} \quad \vec{r} = x\,\hat{i} + y\,\hat{j}$$

In magnitude, $|\vec{r}|$ or $r = \sqrt{x^2 + y^2}$

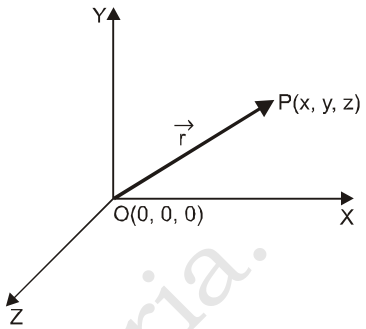


Fig. 1.7. Position vector in three dimensions

If the position of a point P is chosen with reference to the origin of the three-dimensional rectangular co-ordinate system as shown in Fig. 1.7, then the position vector is given by,

$$\vec{r} = \vec{x} + \vec{y} + \vec{z}$$

or $$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

The magnitude or modulus of $\vec{r}$ *is given by r or* $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

1.5. EQUALITY OF VECTORS

Two vectors are said to be ***equal*** *if they have the same magnitude and same direction.* In Fig. 1.8, three equal vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ have been represented. The equality of vectors is represented as follows:

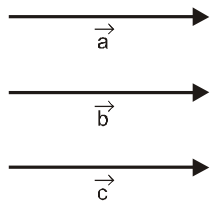


Fig. 1.8. Equal vectors

$$\vec{a} = \vec{b} = \vec{c}$$

Since the three vectors are pointing in the same direction,

$$\therefore \qquad \hat{a} = \hat{b} = \hat{c}$$

Also, since the three vectors have equal magnitudes,

$$\therefore \qquad |\vec{a}| = |\vec{b}| = |\vec{c}|$$

If the scales selected for the representation of three vectors are the same, then three equal vectors are represented by three arrows of equal lengths, pointing in the same direction.

1.6. THE ZERO VECTOR AND ITS PROPERTIES

Zero vector *or* ***null vector*** *is a vector which has zero magnitude and an arbitrary direction.* It is represented by $\vec{0}$.

If $\mu = -\lambda$, then the vector $(\lambda + \mu)\vec{a}$ is equal to $\vec{0}$.

If we multiply a vector by zero, what do we get? The answer is obviously zero vector. Now, let us consider a vector $(\vec{a} + \vec{b})$. If $\vec{b} = -\vec{a}$, then $\vec{a} + \vec{b} = \vec{0}$.

(*i*) The displacement of a ball thrown up and received back by the thrower is a zero vector.

(*ii*) The velocity vector of a stationary body is a zero vector.

1.7. ADDITION OR COMPOSITION OF VECTORS

*The process of adding two or more than two vectors is called **'addition or composition of vectors'.***

When two or more than two vectors are added, we get a single vector called resultant vector.

*The **resultant** of two or more than two vectors is a single vector which produces the same effect as the individual vectors together produce.*

Following three laws have been evolved for the addition of vectors.

(*i*) Triangle law of vectors (*for addition of two vectors*)

(*ii*) Parallelogram law of vectors (*for addition of two vectors*)

(*iii*) Polygon law of vectors (*for addition of more than two vectors*).

Triangle Law of Vectors

Let a particle be at the points A, B and C at three successive times t, t' and t'' respectively. $\vec{AB}$ is the displacement vector from time t to t'. $\vec{BC}$ is the displacement vector from time t' to time t''. The total displacement vector $\vec{AC}$ is the sum or the resultant of individual displacement vectors $\vec{AB}$ and $\vec{BC}$.

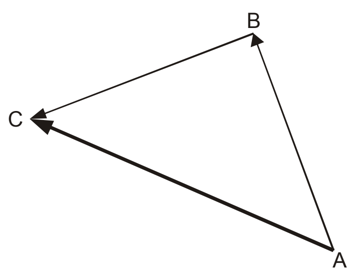


Fig. 1.9. Triangle law of vectors

$$\therefore \quad \vec{AC} = \vec{AB} + \vec{BC}$$

This leads us to the following law known as *triangle law of vectors.* This law is used for the addition of two vectors.

If two vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant is represented completely, both in magnitude and direction, by the third side of the triangle taken in the opposite order.

Suppose we have to add two vectors $\vec{P}$ and $\vec{Q}$ as shown in Fig. 1.10 (*a*). Now, displace $\vec{Q}$ parallel to itself in such a way that the tail of $\vec{Q}$ touches the tip of $\vec{P}$. Complete the triangle to get a new vector $(\vec{P} + \vec{Q})$ running straight from the tail of $\vec{P}$ to the tip of $\vec{Q}$. According to triangle law of vectors, this new vector is the resultant $\vec{R}$ of the given vectors $\vec{P}$ and $\vec{Q}$ such that,

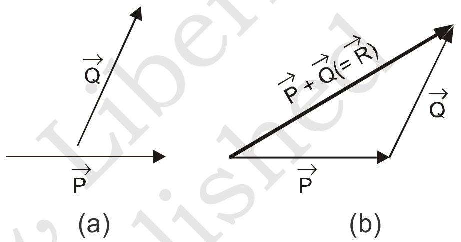


Fig. 1.10. Triangle law of vectors (Graphical method for addition of vectors)

$$\vec{R} = \vec{P} + \vec{Q}.$$

Triangle law of vectors is applicable to triangle of any shape.

It follows from triangle law of vectors that if three vectors are represented by the three sides of a triangle taken in order, then their resultant is zero. Thus, if three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ can be represented completely by the three sides of a triangle taken in order, then their vector sum is zero.

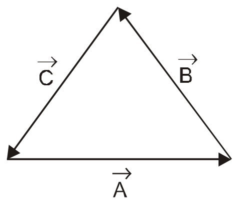


Fig. 1.11

$$\therefore \qquad \vec{A} + \vec{B} + \vec{C} = \vec{0}$$

Parallelogram Law of Vectors

Consider two vectors $\vec{P}$ and $\vec{Q}$ as shown in Fig. 1.12 (*a*). Displace $\vec{Q}$ parallel to itself till the tail of $\vec{Q}$ touches the tail of $\vec{P}$.

Complete the parallelogram as shown in Fig. 1.12 (*b*). Applying triangle law of vectors to the vector triangle OAC, we get

$$\vec{OA} + \vec{AC} = \vec{OC}$$

or $$\vec{P} + \vec{Q} = \vec{R}$$

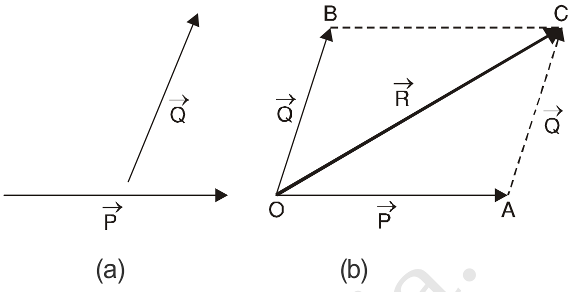


Fig. 1.12. Parallelogram law of vectors

So, we conclude that if two vectors are represented completely by the two adjacent sides, of a parallelogram, drawn from a point, then the diagonal of the parallelogram drawn through that point gives the resultant vector. This is parallelogram law of vectors. It is stated as follows:

"If two vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, then the resultant is represented completely both in magnitude and direction by the diagonal of the parallelogram passing through that point."

In Fig. 1.13, two vectors $\vec{P}$ and $\vec{Q}$ are completely represented by the two sides OA and OB respectively of a parallelogram. Then, according to parallelogram law of vectors, the diagonal OC of the parallelogram will give the resultant $\vec{R}$ such that $\vec{R} = \vec{P} + \vec{Q}$.

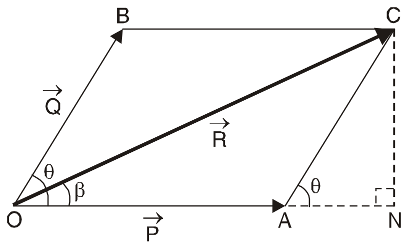


Fig. 1.13. Addition of vectors by parallelogram law of vectors

Let us analytically calculate the magnitude and direction of the resultant vector $\vec{R}$.

Let θ be the angle between two given vectors $\vec{P}$ and $\vec{Q}$. From C, drop a perpendicular CN on OA (produced). In the right-angled ΔANC,

$$\sin\theta = \frac{CN}{AC} \text{ or } CN = AC \sin\theta$$

or $\quad CN = Q \sin\theta \qquad [\because \; AC = OB = Q] \qquad ...(1)$

Also, $\cos\theta = \dfrac{AN}{AC}$ or $AN = AC\cos\theta = Q\cos\theta$

Now, $ON = OA + AN = P + Q\cos\theta \qquad ...(2)$

Considering the right-angled ΔONC,

$$OC^2 = ON^2 + CN^2$$

or $$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

[From (2) and (1)]

or $$R^2 = P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$

$$= P^2 + Q^2\cos^2\theta + Q^2\sin^2\theta + 2PQ\cos\theta$$

$$= P^2 + Q^2(\cos^2\theta + \sin^2\theta) + 2PQ\cos\theta$$

$\therefore$ $$R^2 = P^2 + Q^2 + 2PQ\cos\theta \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

or $$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad ...(3)$$

which is the required expression for the magnitude of the resultant of two vectors $\vec{P}$ and $\vec{Q}$ inclined to each other at an angle θ. Equation (3) is known as the **law of cosines.**

Let β be the angle which the resultant $\vec{R}$ makes with $\vec{P}$.

Then, $$\tan\beta = \frac{CN}{ON} \qquad \text{(in } rt.\ \angle d\ \Delta ONC)$$

or $$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta} \qquad \text{[from (2) and (1)]} \qquad ...(4)$$

or $$\beta = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$$

which gives the direction of the resultant vector.

SPECIAL CASES

Case I. ***When the given vectors $\vec{P}$ and $\vec{Q}$ act in the same direction***

In this case, $\theta = 0°$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos 0°} \quad \text{[from equation (3)]}$$

$$= \sqrt{P^2 + Q^2 + 2PQ} \quad [\because \cos 0° = 1]$$

$$= \sqrt{(P+Q)^2} = P + Q$$

or $$|\vec{R}| = |\vec{P}| + |\vec{Q}|$$

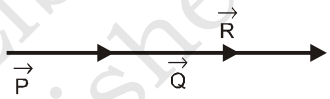


Fig. 1.14

So, *the magnitude of the resultant vector is equal to the sum of the magnitudes of the given vectors.*

Also, $$\tan\beta = \frac{Q\sin 0°}{P + Q\cos 0°} \quad \text{[from equation (4)]}$$

or $$\tan\beta = 0 \quad [\because \sin 0° = 0]$$

$$\therefore \quad \beta = 0°$$

So, *the resultant vector points in the direction of the given vectors.*

Case II. ***When the given vectors $\vec{P}$ and $\vec{Q}$ act at right angles to each other***

In this case, $\theta = 90°$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos 90°}$$

or $$R = \sqrt{P^2 + Q^2} \quad [\because \cos 90° = 0]$$

or $$|\vec{R}| = \sqrt{|\vec{P}|^2 + |\vec{Q}|^2}$$

Also, $$\tan\beta = \frac{Q\sin 90°}{P + Q\cos 90°}$$

or $$\tan\beta = \frac{Q}{P} \quad [\because \sin 90° = 1]$$

or $$\beta = \tan^{-1}\left(\frac{Q}{P}\right)$$

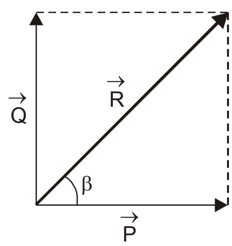


Fig. 1.15

If P = Q, then $R = \sqrt{P^2 + P^2}$ or $R = \sqrt{2P^2} = \sqrt{2}\,P$

Also, in this case, $\tan\beta = \frac{P}{P} = 1$ or $\beta = 45°$

Case III. *When the given vectors $\vec{P}$ and $\vec{Q}$ act in opposite directions*

In this case, $\theta = 180°$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos 180°}$$

$$= \sqrt{P^2 + Q^2 - 2PQ} \quad [\because \cos 180° = -1]$$

$$= \sqrt{(P-Q)^2}$$

$$\therefore \quad R = \pm(P - Q) = P - Q \text{ or } Q - P$$

or $$|\vec{R}| = |\vec{P}| \sim |\vec{Q}|$$

[$|\vec{P}| \sim |\vec{Q}|$ implies positive difference between $|\vec{P}|$ and $|\vec{Q}|$.]

So, *the magnitude of the resultant vector is equal to the positive difference of the magnitudes of the given vectors.*

Also, $$\tan\beta = \frac{Q\sin 180°}{P + Q\cos 180°}$$

or $$\tan\beta = 0$$

$$\therefore \quad \beta = 0° \text{ or } 180°$$

When $|\vec{P}| > |\vec{Q}|$, then $\beta = 0°$. [Fig. 1.16]

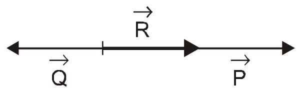


Fig. 1.16

When $|\vec{P}| < |\vec{Q}|$, then $\beta = 180°$. [Fig. 1.17]

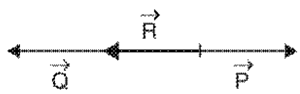


Fig. 1.17

Clearly, *the resultant vector acts in the direction of the bigger of the two vectors.*

Illustrations of Parallelogram Law of Vectors

1. Flight of a bird. When a bird flies, its wings W_1 and W_2 push the air downwards with forces F_1 and F_2 respectively. The air offers equal and opposite reactions R_1 and R_2 in accordance with Newton's third law of motion. According to parallelogram law of vectors, the resultant R of R_1 and R_2 acts on the bird in the upward direction [Fig. 1.18]. This helps the bird to fly upward.

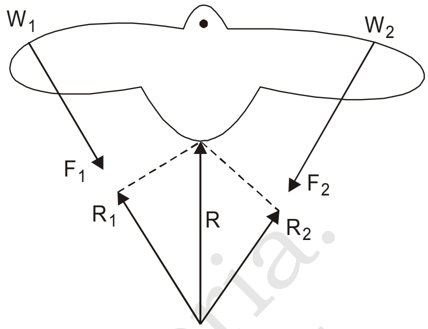


Fig. 1.18

2. Working of a sling. A sling is a Y-shaped metallic or wooden frame to which a rubber band is attached. Tensions T_1 and T_2 are produced in the rubber band when a stone held on the rubber band is pulled [Fig. 1.19]. The resultant of T_1 and T_2 is T in accordance with parallelogram law of vectors. When the stone is released, it moves under the action of T with high speed.

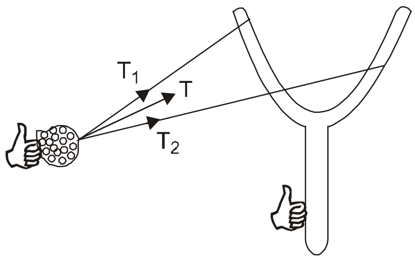


Fig. 1.19

Polygon Law of Vectors

Polygon law of vectors is used for the addition of more than two vectors.

Consider four vectors $\vec{P}$, $\vec{Q}$, $\vec{S}$ and $\vec{T}$ as shown in Fig. 1.20 (*a*). Displace $\vec{Q}$ parallel to itself till the tail of $\vec{Q}$ touches the tip of $\vec{P}$. Similarly, displace $\vec{S}$ parallel to itself till the tail of $\vec{S}$ touches the tip of $\vec{Q}$. Again, displace $\vec{T}$ parallel to itself so that its tail touches the tip of $\vec{S}$. Now a vector $\vec{R}$ running straight from the tail of $\vec{P}$ to the tip of $\vec{T}$ will be the resultant of $\vec{P}$, $\vec{Q}$, $\vec{S}$ and $\vec{T}$.

This is polygon law of vectors stated as follows :

"If a number of vectors, acting simultaneously at a point, can be represented both in magnitude and direction by the sides of an open

convex polygon taken in the same order, then the resultant is represented completely in magnitude and direction by the closing side of the polygon, taken in the opposite order.”

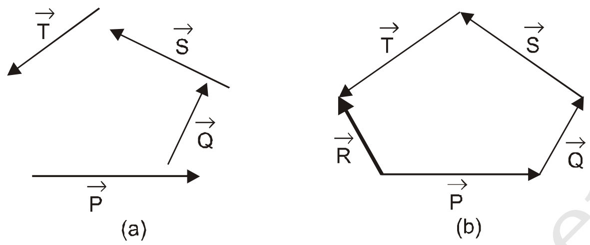


Fig. 1.20. Polygon law of vectors

1.8. PROPERTIES OF VECTOR ADDITION

A quantity can be a vector only if it obeys the laws of vector addition.

Following are the important properties of vector addition.

(*i*) Vectors of the same nature alone can be added. A force vector can be added to force vector only. It cannot be added to displacement vector.

(*ii*) Vector addition is commutative. *The sum of the vectors remains the same in whatever order they may be added.*

According to commutative law of vector addition,

$$\vec{a} + \vec{b} + \vec{c} + \ldots\ldots = \vec{b} + \vec{a} + \vec{c} + \ldots\ldots = \vec{c} + \vec{a} + \vec{b} + \ldots\ldots$$

The result of vector addition does not depend on the order in which the vector sum is written.

(*iii*) Vector addition is distributive.

According to distributive law of vector addition,

$$\lambda(\vec{a} + \vec{b}) = \lambda\vec{a} + \lambda\vec{b}$$

(*iv*) Vector addition is associative. *The sum of the vectors remains the same in whatever grouping they are added.*

According to associative law of vector addition,

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

1.9 EQUILIBRANT VECTOR

Equilibrant vector *is a single vector which balances two or more than two vectors acting simultaneously at a point.*

The equilibrant and the resultant vectors are equal in magnitude and opposite in direction.

Example 1. *Resultant of two vectors* $\vec{a}$ *and* $\vec{b}$ *inclined at an angle* θ *is* $\vec{c}$. *Calculate* θ.

Given : $|\vec{a}| = |\vec{b}| = |\vec{c}|$

Solution. $c^2 = a^2 + b^2 + 2ab\cos\theta$

or $c^2 = c^2 + c^2 + 2c^2\cos\theta$ $\quad[\because\ a = b = c]$

or $c^2 = 2c^2(1 + \cos\theta)$

or $1 + \cos\theta = \dfrac{1}{2}$ $\quad \cos 120° = -\dfrac{1}{2}$

or $\cos\theta = -\dfrac{1}{2}$ or $\theta = \mathbf{120°}$

1.10. SUBTRACTION OF VECTORS

Subtraction of a vector $\vec{B}$ *from a vector* $\vec{A}$ *is the addition of vector* $(-\vec{B})$ *to vector* $\vec{A}$.

The subtraction of two vectors often becomes necessary in connection with velocities and accelerations. It is, of course, not very common in the case of forces.

The process of subtracting one algebraic quantity from another is equivalent to adding the negative of the quantity to be subtracted.

$$a - b = a + (-b)$$

In the same manner, the process of subtracting one vector quantity from the other is equivalent to adding vectorially the negative of the vector to be subtracted. Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

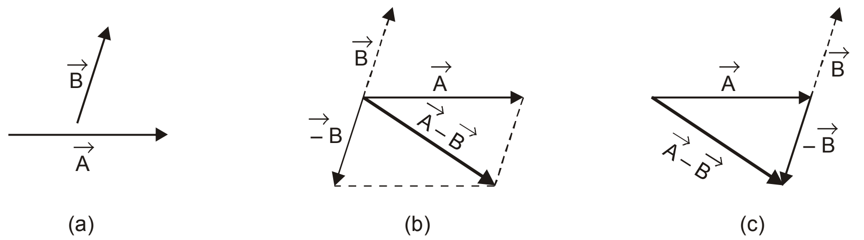


Fig. 1.21. Subtraction of vectors

1.11. PROJECTILE

A body which is in flight through the atmosphere under the influence of gravity alone without being propelled by any fuel is called a ***projectile.***

Examples:

(*i*) A bomb released from an aeroplane in level flight.

(*ii*) A bullet fired from a gun.

(*iii*) A javelin thrown by an athlete.

(*iv*) An arrow released from bow.

(*v*) A stone thrown horizontally from the top of a building.

The path followed by a projectile is called ***trajectory.***

The motion of a projectile is a two-dimensional motion.

1.12 TWO TYPES OF PROJECTILES

Following are the two types of projectiles.

(*i*) Horizontal Projectile. If a body is projected horizontally from a certain height with a certain velocity, then the body is called a horizontal projectile.

(*ii*) Oblique Projectile. If a body is projected at a certain angle with the horizontal, then the body is called an oblique projectile.

1.13. PRINCIPLE OF PHYSICAL INDEPENDENCE OF MOTIONS

The motion of a projectile is a two-dimensional motion. So, it can be discussed in two parts.

(*i*) horizontal motion **(*ii*)** vertical motion.

These two motions take place independent of each other. This is called the principle of physical independence of motions.

At any instant, the velocity of a projectile has two components **(*i*)** horizontal component **(*ii*)** vertical component.

The horizontal component remains unchanged throughout the flight. The vertical component is continuously affected by the force of gravity. Thus, while the horizontal motion is a uniform motion, the vertical motion is a uniformly accelerated motion.

1.14. HORIZONTAL PROJECTILE

(*i*) Nature of Trajectory. Consider a projectile thrown horizontally from a point O, with horizontal velocity $\vec{u}$, at a certain height above the ground.

Through the point O, take two axes—X-axis and Y-axis. Let x and y be the horizontal and vertical distances respectively covered by the projectile in time t. At time t, the projectile is at P (Fig. 1.22).

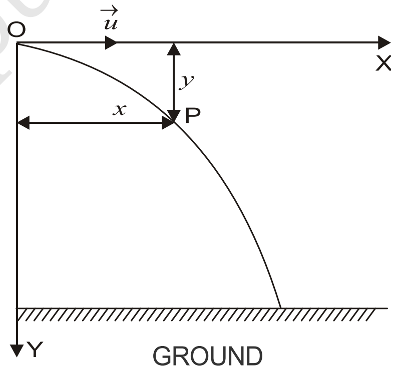


Fig. 1.22. Trajectory of horizontal projectile

The horizontal motion of the projectile is uniform motion. This is because the only force acting on the projectile is force of gravity. This force acts in the vertically downward direction and its horizontal component is zero.

Using $x = x_0 + u_x t + \frac{1}{2} a_x t^2$, we get

$$x = 0 + ut + 0 \quad \text{or} \quad x = ut \quad \text{or} \quad t = \frac{x}{u} \qquad \ldots(1)$$

The **vertical motion** of the projectile is controlled by force of gravity and is an accelerated motion. The initial velocity u_y in the vertically downward direction is zero. Since Y-axis is taken downwards, therefore, the downward direction will be regarded as positive direction. So, the acceleration a_y of the projectile is + g.

Using $$y = y_0 + u_y t + \frac{1}{2} a_y t^2, \text{ we get}$$

$$\therefore \quad y = 0 + 0 + \frac{1}{2} g t^2 \quad \text{or} \quad y = \frac{1}{2} g t^2 \qquad \text{...(2)}$$

Combining (1) and (2), we get

$$y = \frac{1}{2} g \left(\frac{x}{u}\right)^2 \quad \text{or} \quad y = \frac{g}{2u^2} x^2 \quad \text{or} \quad y = kx^2 \qquad \text{...(3)}$$

where $k \left(= \frac{g}{2u^2}\right)$ is a constant.

Equation (3) is a second degree equation in x and a first degree equation in y. This is the equation of a parabola.

In the study of projectile motion, both position and time are measured from 'O'. $\therefore \quad x_0 = y_0 = 0.$

CONCLUSION
A body thrown horizontally from a certain height above the ground follows a parabolic trajectory till it hits the ground.

(*ii*) Time of Flight (T). *It is the time of descent of the projectile from the point of projection to the ground. It is the total time for which the projectile is in flight.*

Let h be the vertical height of the point of projection above the ground.

Considering vertically downward motion,

$$y = y_0 + u_y t + \frac{1}{2} a_y t^2$$

Putting values, $h = 0 + 0 + \frac{1}{2} g \mathrm{T}^2$ or $\mathrm{T} = \sqrt{\frac{2h}{g}}$

(*iii*) **Horizontal Range (R).** *It is the horizontal distance travelled by the projectile during the time of flight.*

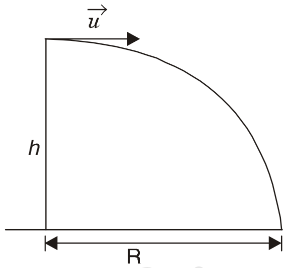


Fig. 1.23

Using $x = x_0 + u_x t + \frac{1}{2} a_x t^2$, we get

$$R = 0 + uT + 0$$

$$= uT = u\sqrt{\frac{2h}{g}}.$$

1.15. TRAJECTORY OF AN OBLIQUE PROJECTILE

Consider a projectile thrown with velocity u at an angle θ with the horizontal (Fig. 1.24). The velocity u can be resolved into two rectangular components (*i*) $u \cos \theta$ along X-axis and (*ii*) $u \sin \theta$ along Y-axis. The motion of the projectile is a two-dimensional motion. It can be supposed to be made up of two motions-horizontal motion (along X-axis) and vertical motion (along Y-axis). The **horizontal motion** of the projectile is uniform motion. This is because the only force acting on the projectile is the force of gravity. This force acts in the vertically downward direction and its horizontal component is zero. Thus, the equations of motion of the projectile for the horizontal direction are simply the equations of uniform motion in a straight line. The horizontal motion takes place with constant velocity $u \cos \theta$. If x be the horizontal distance covered in time t, then

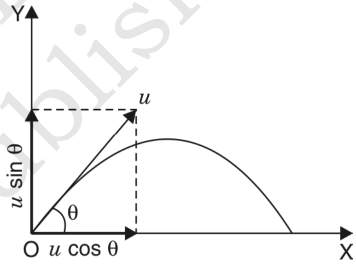


Fig. 1.24. Trajectory of an oblique projectile

$$x = (u \cos \theta)\, t$$

or

$$t = \frac{x}{u \cos \theta} \quad \ldots(1)$$

The **vertical motion** of the projectile is controlled by the force of gravity. The projectile increases its height up to a maximum where its vertical velocity v_y becomes zero. After this, the projectile reverses its vertical direction and returns to earth striking the ground with a speed u which is the same as the initial speed of the projectile.

Let y be the vertical distance covered by the projectile in time t. Let us now consider the vertical motion of the projectile.

$$u_y = u \sin \theta,\ a_y = -g,\ 't' = t$$

We know that $$y = u_y t + \frac{1}{2} a_y t^2$$

Substituting values, $y = u \sin \theta\ t - \frac{1}{2} g t^2$

Using equation (1), $$y = u \sin \theta \left(\frac{x}{u \cos \theta}\right) - \frac{1}{2} g \left(\frac{x}{u \cos \theta}\right)^2$$

or $$\boxed{y = x \tan \theta - \frac{g x^2}{2u^2 \cos^2 \theta}} \qquad \text{...(2)}$$

This is a first degree equation in y and a second degree equation in x. This is the equation of a *parabola. So, the path followed by the projectile, *i.e.*, **the trajectory of the projectile is parabolic.**

It is clear from equation (2) that the trajectory is completely known if u and θ are known. It should be kept in mind that equation (2) is valid only if θ lies between 0 and $\pi/2$.

1.16. MAXIMUM HEIGHT

It is the maximum height to which a projectile rises above the horizontal plane of projection. It is denoted by $h_{max.}$ or H. It is also known as vertical range.

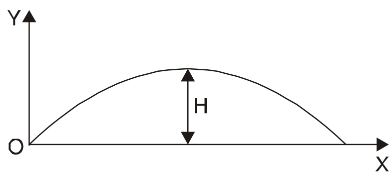


Fig. 1.25. Maximum height of an oblique projectile

In order to calculate the maximum height H, we make use of the fact that the velocity v_y of the projectile at the maximum height is zero. If t_1 be the time taken by the projectile to reach maximum height, then using equation $v_y = u_y + a_y t$, we get

$$0 = u \sin \theta - g t_1 \quad \text{or} \quad g t_1 = u \sin \theta \quad \text{or} \quad t_1 = \frac{u \sin \theta}{g}$$

When $'t' = t_1$, y = H.

$$y_0 = 0,\ u_y = u \sin \theta,\ a_y = -g$$

*The general equation of a parabola which passes through the origin is $y = ax - bx^2$.

Using relation $y = y_0 + u_y\, t + \frac{1}{2}\, a_y\, t^2$, we get

$$H = (u \sin \theta)\, t_1 - \frac{1}{2} g t_1^2$$

or $$H = u \sin \theta \times \frac{u \sin \theta}{g} - \frac{1}{2}\, g \left(\frac{u \sin \theta}{g}\right)^2$$

or $$H = \frac{u^2 \sin^2 \theta}{g} - \frac{u^2 \sin^2 \theta}{2g} \quad \text{or} \quad \boxed{H = \frac{u^2 \sin^2 \theta}{2g}}$$

1.17. TIME OF FLIGHT

Time of flight *is the total time taken by the projectile to return to the same level from where it was thrown.* It is the total time for which the projectile is in flight.

Time of flight is equal to twice the time taken by the projectile to reach the maximum height. This is because the time of ascent is equal to the time of descent. This fact is also clear from the symmetry of the curve.

Time of flight, $T = 2t$

where *t is the time taken by the projectile to reach maximum height.

Now, $v_y = 0,\ a_y = -g,\ u_y = u \sin \theta$

We know that $v_y = u_y + a_y t$

Substituting values, $0 = u \sin \theta - gt$ or $gt = u \sin \theta$

or $$t = \frac{u \sin \theta}{g}$$

$\therefore$ $$\boxed{T = \frac{2u \sin \theta}{g}}$$

1.18. HORIZONTAL RANGE

Horizontal range *is the total horizontal distance from the point of projection to the point where the projectile comes back to the plane of projection.* It is denoted by R.

* This is the time of ascent of the projectile.

In order to calculate horizontal range R, we shall consider horizontal motion of the projectile. The horizontal motion is uniform motion. It takes place with constant velocity $u \cos \theta$.

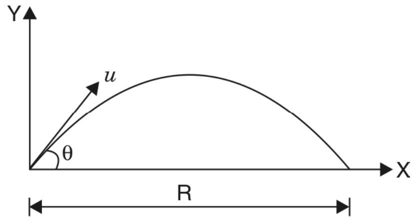


Fig. 1.26. Horizontal range of an oblique projectile

$$\therefore \quad R = u \cos \theta \times \text{time of flight}$$

$$= u \cos \theta \times \frac{2u \sin \theta}{g}$$

or $$R = \frac{u^2 (2 \sin \theta \cos \theta)}{g}$$

or $$\boxed{R = \frac{u^2 \sin 2\theta}{g}} \qquad (\because 2 \sin \theta \cos \theta = \sin 2\theta)$$

1.19. MAXIMUM HORIZONTAL RANGE

For a given velocity of projection and at a given place, the value of R will be maximum when the value of sin 2θ is maximum *i.e.,* 1.

For R to be maximum, $\sin 2\theta = 1$ (maximum value)

or $$\sin 2\theta = \sin 90°$$

or $$\theta = 45°$$

So, for a given velocity, the angle of projection for maximum range is 45°, *i.e.,* $\frac{x}{4}$.

Maximum horizontal range, $\boxed{R_{max.} = \frac{u^2}{g}}$

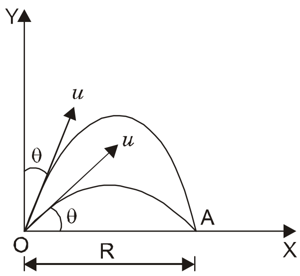


Fig. 1.27. Two angles of projection for the same range

1.20. TWO ANGLES OF PROJECTION FOR THE SAME RANGE

Again, $$R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin (180° - 2\theta)}{g} \qquad [\because \sin (180° - 2\theta) = \sin 2\theta]$$

or $$R = \frac{u^2 \sin 2(\theta)}{g}$$

$$= \frac{u^2 \sin 2(90° - \theta)}{g}$$

This shows that there are two angles of projection for the same horizontal range *i.e.,* θ and (90° – θ) with the horizontal. The projectile will cover the same horizontal range whether it is thrown at an angle θ or (90° – θ) with the horizontal.

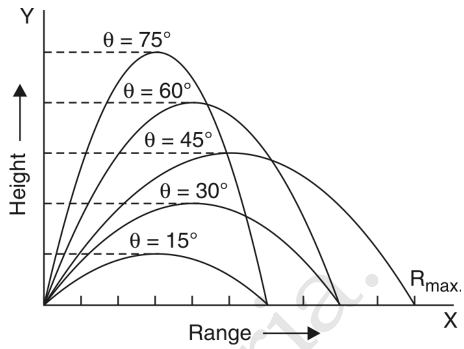


Fig. 1.28. R is same for θ = 15° and 75°. Again, R is same for θ = 30° and 60°. R is maximum for θ = 45°

Example 2. *A projectile is thrown at an angle θ with the horizontal with kinetic energy E. Calculate the potential energy at the topmost point of the trajectory.*

Solution. Potential energy at the topmost point of the trajectory

$$= mg\,h_{max.} = mg\,\frac{u^2 \sin^2\theta}{2g}$$

$$= \left(\frac{1}{2}mu^2\right)\sin^2\theta = \mathbf{E\,sin^2\,\theta}$$

Example 3. *A projectile is thrown with an initial velocity of $x\hat{i} + y\hat{j}$. The range of the projectile is twice the maximum height of the projectile. Calculate $\frac{y}{x}$.*

Solution. $\frac{u^2 \sin 2\theta}{g} = 2\,\frac{u^2 \sin^2\theta}{2g}$

or $2u^2 \sin\theta \cos\theta = u^2 \sin^2\theta$

or $2(u \sin\theta)(u \cos\theta) = (u \sin\theta)(u \sin\theta)$

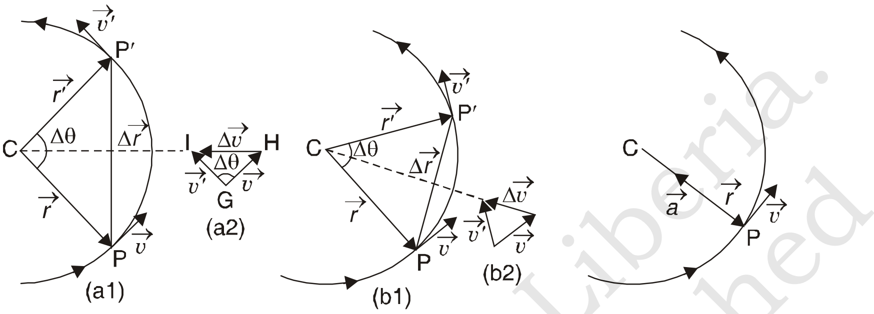
But $u \sin\theta = y$ and $u \cos\theta = x$ (given)

∴ $2yx = y^2$ or $2x = y$ or $\frac{y}{x} = \mathbf{2}$

1.21. UNIFORM CIRCULAR MOTION

When an object follows a circular path at a constant speed, the motion of the object is called uniform circular motion. The word "uniform" refers to the speed which is uniform (constant) throughout the motion.

Consider an object moving with uniform speed v in a circle of radius R as shown in Fig. 1.29. Since the velocity of the object is changing continuously in direction, therefore, the object undergoes acceleration. Let us find the magnitude and direction of this acceleration.



(*a*) (*b*) (*c*)

Fig. 1.29. Velocity and acceleration of an object in uniform circular motion. The time interval Δt decreases from (*a*) to (*c*) where it is zero. The acceleration is directed, at each point of the path, towards the centre of the circle

Let $\vec{r}$ and $\vec{r'}$ be the position vectors and $\vec{v}$ and $\vec{v'}$ the velocities of the object when it is at point P and P′ as shown in Fig. 1.29(*a*). By definition, velocity at a point is along the tangent at that point in the direction of motion. The velocity vectors $\vec{v}$ and $\vec{v'}$ are as shown in Fig. 1.29(*a*). $\Delta\vec{v}$ is obtained in Fig. 1.29(*b*) using the triangle law of vector addition. Since the path is circular, $\vec{v}$ is perpendicular to $\vec{r}$ and so is $\vec{v'}$ to $\vec{r'}$. Therefore, $\Delta\vec{v}$ is perpendicular to $\Delta\vec{r}$. Since average acceleration is along $\Delta\vec{v}$ $\left(\vec{a}_{av.} = \dfrac{\Delta\vec{v}}{\Delta t}\right)$, the average acceleration $\vec{a}_{av.}$ is perpendicular to $\Delta\vec{r}$. If we place $\Delta\vec{v}$ on the line that bisects the angle between $\vec{r}$ and $\vec{r'}$, we see that it is directed towards the centre of the circle. Figure 1.29(*b*) shows the same quantities for smaller time interval. $\Delta\vec{v}$ and hence $\vec{a}_{av.}$ is again directed towards the centre. In Fig. 1.29(*c*), $\Delta t \to 0$ and the average acceleration becomes the instantaneous acceleration. It is directed towards the centre.

The magnitude of $\vec{a}$ is, by definition, given by

$$|\vec{a}| = \lim_{\Delta t \to 0} \frac{|\Delta\vec{v}|}{\Delta t}$$

Let the angle between position vectors $\vec{r}$ and $\vec{r'}$ be $\Delta\theta$. Since the velocity vectors $\vec{v}$ and $\vec{v'}$ are always perpendicular to the position vectors, the angle between them is also $\Delta\theta$. Therefore, the triangle CPP′ formed by the position vectors and the triangle GHI formed by the velocity vectors $\vec{v}$, $\vec{v'}$ and $\Delta\vec{v}$ are similar [Fig. 1.29(*a*)]. Therefore, the ratio of the base-length to side-length for one of the triangles is equal to that of the other triangle. That is:

$$\frac{|\Delta\vec{v}|}{v} = \frac{|\Delta\vec{r}|}{R} \quad \text{or} \quad |\Delta\vec{v}| = v\frac{|\Delta\vec{r}|}{R}$$

Therefore, $$|\vec{a}| = \lim_{\Delta t \to 0} \frac{|\Delta\vec{v}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v|\Delta\vec{r}|}{R\,\Delta t} = \frac{v}{R}\lim_{\Delta t \to 0} \frac{|\Delta\vec{r}|}{\Delta t}$$

If Δt is small, $\Delta\theta$ will also be small and then arc PP′ can be approximately taken to be $|\Delta\vec{r}|$.

$$|\Delta\vec{r}| \cong v\Delta t$$

$$\frac{|\Delta\vec{r}|}{\Delta t} \cong v$$

or $$\lim_{\Delta t \to 0} \frac{|\Delta\vec{r}|}{\Delta t} = v$$

Therefore, the centripetal acceleration a_c is:

$$a_c = \left(\frac{v}{R}\right)v = \frac{v^2}{R}$$

Thus, the acceleration of an object moving with speed v in a circle of radius R has a magnitude $\frac{v^2}{R}$ and is always **directed towards the centre.** This is why this acceleration is called **centripetal acceleration** (a term proposed by Newton). Since v and R are constant, the magnitude of the centripetal acceleration is also constant. However, the direction changes—pointing always towards the centre. Therefore, a centripetal acceleration is not a constant vector.

As the object moves from P to P′ in time Δt (= $t' - t$), the line CP (Fig. 1.29.) turns through an angle $\Delta\theta$ as shown in the figure. $\Delta\theta$ is called angular distance. We define the angular speed ω (Greek letter omega) as the time rate of change of angular displacement.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Now, if the distance travelled by the object during the time Δt is Δs, *i.e.,* PP′ is Δs, then:

$$v = \frac{\Delta s}{\Delta t}$$

but $\Delta s = R\,\Delta\theta$. Therefore:

$$v = R\frac{\Delta\theta}{\Delta t} = R\omega$$

We can express centripetal acceleration a_c in terms of angular speed:

$$a_c = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$$

$$a_c = \omega^2 R$$

The time taken by an object to make one revolution is known as its time period T and the number of revolutions made in one second is called its frequency $\nu\left(=\frac{1}{T}\right)$. However, during this time the distance moved by the object is, $s = 2\pi R$.

Therefore, $v = \frac{2\pi R}{T} = 2\pi R\nu$

In terms of frequency ν, we have

$$\omega = 2\pi\nu$$

$$v = 2\pi R\nu$$

$$a_c = 4\pi^2\,\nu^2 R.$$

Example 4. *A constant torque is acting on a wheel. If starting from rest, the wheel makes n rotations in t second, show that the angular acceleration is given by* $\alpha = \frac{4\pi n}{t^2}$ *rad* s^{-2}.

Solution. Since the wheel starts from rest, therefore, the initial angular velocity ω_0 is zero.

Number of rotations in t second = n

Angular displacement in time t, $\theta = 2\pi n$

Now, $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

or $$2\pi n = 0 + \frac{1}{2}\alpha t^2 \quad \text{or} \quad \alpha = \frac{4\pi n}{t^2} \ \textbf{rad s}^{-2}$$

1.22. OSCILLATORY MOTION

If a body moves back and forth repeatedly about a mean position, it is said to possess ***oscillatory*** *or* ***vibratory motion.***

Very often the body undergoing periodic motion has an equilibrium position somewhere inside its path. When the body is at this position, no net external force acts on it. Therefore, if it is left there at rest, it remains there forever. If the body is given a small displacement from this position, a force comes into play which tries to bring the body back to the equilibrium point, giving rise to **oscillations** or **vibrations**. For example, a ball placed in a bowl will be in equilibrium at the bottom. If displaced a little from the point, it will perform oscillations in the bowl.

Examples. ***(i)*** Motion of the pendulum of a wall clock. ***(ii)*** Vibrations of the wire of a '*sitar*'. ***(iii)*** Vibrations of the drum of a '*tabla*'. ***(iv)*** Oscillations of a mass suspended from a spring. ***(v)*** Motion of liquid in a U-tube when the liquid is once compressed in one limb and then left to itself. ***(vi)*** A weighted test tube floating in a liquid executes oscillatory motion when pressed down and released.

Difference between periodic motion and oscillatory motion. An oscillatory motion is always periodic. A periodic motion may or may not be oscillatory. So, oscillatory motion is merely a special case of periodic motion. As an example, the motion of the planets around the Sun is periodic but not **oscillatory.**

Difference between oscillations and vibrations. There is no significant difference between oscillations and vibrations. When the frequency is small, we use the term "oscillation" (like the oscillation of a branch of a tree). When the frequency is high, we use the term "vibration" (like the vibration of a string of a musical instrument).

Simplest form of oscillatory motion. The simplest form of oscillatory motion is *simple harmonic motion.* This motion arises when the force on the oscillating body is directly proportional to its displacement from the mean position (equilibrium position). At any point in its oscillation, this force is directed towards the mean position.

1.23. SIMPLE HARMONIC MOTION

Simple Harmonic Motion is a motion which is necessarily periodic and oscillatory about a fixed mean position. A particle executing such a motion is always in *stable equilibrium* about its mean position. So, if a particle is disturbed slightly from its mean position, it tends to return to its mean position. The force which tends to bring the particle back to the mean position is called the *restoring force.* The greater the displacement of the particle from the mean position, greater is the restoring force. Thus, ***simple harmonic motion*** *is defined as such an oscillatory motion about a fixed point (mean position) in which the restoring force is always proportional to the displacement from that point and is always directed towards that point.*

If a particle suffers a small displacement x from its mean position, then the magnitude of restoring force F is given by

$$F = -kx \qquad \ldots(1)$$

where k is known as the *force constant.* Its SI unit is N m^{-1}. Its dimensional formula is $[ML^{\circ} T^{-2}]$. The negative sign in equation (1) indicates that the restoring force is directed towards the mean position.

Importance of the study of simple harmonic motion. Any periodic motion can be expressed as the resultant of two or more simple harmonic motions. So, simple harmonic motion is the simplest and most fundamental of all types of periodic motions.

1. In mechanical wave motion, the particles of the medium execute either simple harmonic motion or a combination of simple harmonic motions.
2. The vibrations of the air columns and strings of musical instruments are either simple harmonic or a superposition of simple harmonic motions.

3. The prongs of a vibrating tuning fork oscillate simple harmonically.

1.24. ROTATIONAL MOTION

Consider a rigid body which is so constrained that it cannot have translational motion. The only possible motion of such a rigid body is **rotation.** The line along which the body is fixed is termed as its **axis of rotation.** If you look around, you will come across many examples of rotation about an axis, a ceiling fan, a potter's wheel, a giant wheel in a fair, a merry-go-round and so on [Fig. 1.30(*a*) and (*b*)].

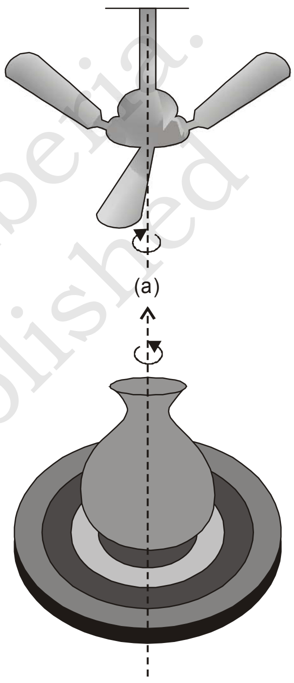


Fig. 1.30. Rotation about a fixed axis (*a*) A ceiling fan (*b*) A potter's wheel

In rotation of a rigid body about a fixed axis, every particle of the body moves in a circle, which lies in a plane perpendicular to the axis and has its centre on the axis. Fig. 1.31 shows the rotational motion of a rigid body about a fixed axis (the *z*-axis of the frame of reference). Let P_1 be a particle of the rigid body, arbitrarily chosen and at a distance r_1 from fixed axis. The particle P_1 describes a circle of radius r_1 with its centre C_1 on the fixed axis. The circle lies in a plane perpendicular to the axis. The figure also shows another

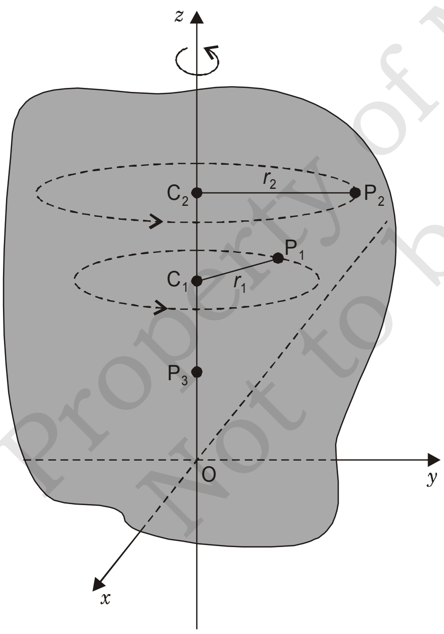


Fig. 1.31. A rigid body rotation about the *z*-axis (Each point of the body such as P_1 or P_2 describes a circle with its centre (C_1 or C_2) on the axis. The radius of the circle (r_1 or r_2) is the perpendicular distance of the point (P_1 or P_2) from the axis. A point on the axis like P_3 remains stationary.)

particle P_2 of the rigid body, P_2 is at a distance r_2 from the fixed axis. The particle P_2 moves in a circle of radius r_2 and with centre C_2 on the axis. This circle, too, lies in a plane perpendicular to the axis. Note that the circles described by P_1 and P_2 may lie in different planes; both these planes, however, are perpendicular to the fixed axis. For any particle on the axis like P_3, $r = 0$. Any such particle remains stationary while the body rotates. This is expected since the axis is fixed.

In some examples of rotation, however, the axis may not be fixed. A prominent example of this kind of rotation is a top spinning in place [Fig. 1.32.]. (We assume that the top does not slip from place to place and so does not have translational motion.) We know from experience that the axis of such a spinning top moves around the vertical through its point of contact with the ground, sweeping out a cone as shown in Fig. 1.32. (This movement of the axis of the top around the vertical is termed **precession.)** Note, the **point of contact of the top with ground is fixed.** The axis of rotation of the top at any instant passes through the point of contact. Another simple example of this kind of rotation is the oscillating table fan or a pedestal fan. You may have observed that the axis of rotation of such a fan has an oscillating (sidewise) movement in a horizontal plane about the vertical through the point at which the axis is pivoted [point O in Fig. 1.33].

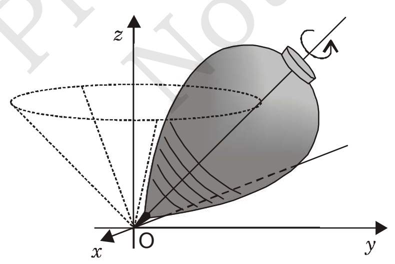


Fig. 1.32. (*a*) A spinning top (The point of contact of the top with the ground, its tip O, is fixed)

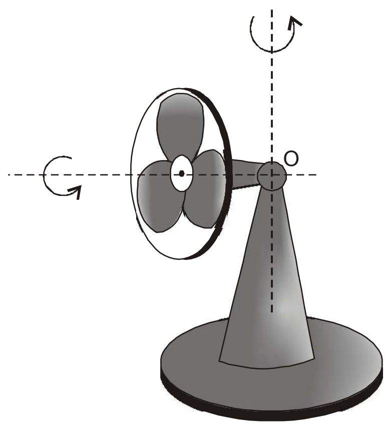


Fig. 1.33. (*b*) An oscillating table fan. The pivot of the fan, point O, is fixed

REVIEW EXERCISES

Do the review exercises in your notebook.

A. Multiple Choice Questions

1. A body moves in a plane so that the displacements along the x and y axes are given by $x = 3t^3$ and $y = 4t^3$. The velocity of the body is

(*a*) $9\ t$ (*b*) $15\ t$

(*c*) $15\ t^2$ (*d*) $25\ t^2$.

2. A particle is travelling along a straight line OX. The distance x (in metre) of the particle from O at a time t is given by $x = 37 + 27t - t^3$ where t is time in second. The distance of the particle from O when it comes to rest is

(*a*) 81 m (*b*) 91 m

(*c*) 101 m (*d*) 111 m.

3. A bullet on penetrating 30 cm into its target loses its velocity by 50%. What additional distance will it penetrate into the target before it comes to rest?

(*a*) 30 cm (*b*) 20 cm

(*c*) 10 cm (*d*) 5 cm.

4. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m s^{-1} where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m s^{-2}, the equation of its trajectory is

(*a*) $4y = 2x - 25x^2$ (*b*) $y = x - 5x^2$

(*c*) $y = 2x - 5x^2$ (*d*) $4y = 2x - 5x^2$

5. The distance x covered by a particle varies with time t as $x^2 = 2t^2 + 6t + 1$. Its acceleration varies with x as

(*a*) x (*b*) x^2

(*c*) x^{-1} (*d*) x^{-3}

B. Fill in the Blanks

1. In the entire path of a projectile, the quantity that remains unchanged is __________ .

2. Among the following, the vector quantity is __________ .

3. If the velocity (in m s^{-1}) of a particle is given by $4.0\hat{i} + 5.0t\hat{j}$, then the magnitude of its acceleration (in m s^{-2}) is __________ .
4. The horizontal range of a projectile is maximum when the angle of projection is __________ .
5. The graph between displacement and time for a particle moving with uniform acceleration is a __________ .

C. Very Short Answer Questions

1. Name a quantity which remains unchanged during the flight of an oblique projectile.
2. At which point of the projectile path, the speed is minimum?
3. Name five physical quantities which change during the motion of an oblique projectile.
4. A body is projected so that it has maximum range R. What is the maximum height reached during the flight?
5. Name two quantities which would be reduced if air resistance is taken into account in the study of motion of oblique projectile.

D. Short Answer Questions

1. A ball is thrown horizontally and at the same time another ball is dropped from the top of a tower. (*i*) Will both the balls hit the ground with the same velocity? (*ii*) Will both the balls reach the ground at the same time?
2. What is the effect of air resistance on the time of flight and horizontal range of the projectile?
3. A projectile of mass m is projected with velocity v at an angle θ with the horizontal. What is the magnitude of the change in momentum of the projectile after time t?
4. The maximum horizontal range of a cannon is 4 km. What is the muzzle velocity of the shell, if g = 10 m s^{-2} ?
5. Why does a tennis ball bounce higher on hills than in plains?

E. Long Answer Questions

1. A motorboat is racing towards north at 25 km h^{-1} and the water current in that region is 10 km h^{-1} in the direction of 60° east of south. Find the resultant velocity of the boat.

2. Two vectors acting in opposite directions have a resultant of 10 units. If they act at right angles to each other, the resultant is 50 units. Calculate the magnitudes of the two vectors.

3. A car travelling at 20 m s^{-1} due north along the highway makes a right turn on to a side road that heads due east. It takes 50 s for the car to complete the turn. At the end of 50 second, the car has a speed of 15 m s^{-1} along the side road. Determine the magnitude of average acceleration over the 50 second interval.

4. A child pulls a rope attached to a stone with a force of 60 N. The rope makes an angle of 40° to the ground.

   (a) Calculate the effective value of the pull tending to move the stone along the ground.

   (b) Calculate the force tending to lift the stone vertically.

5. Referred to two rectangular axes, the three successive displacement vectors have components of + 2.4 m, + 0.5 m ; – 4.6 m, + 3.3 m ; and – 2.8 m, – 15.8 m. Calculate the components of the resultant displacement. What is the magnitude of the resultant?

SEMESTER-1 (Period-II)

P11CH2

TOPIC

2

Composition and Resolution of Force

2.1. PROPERTIES OF VECTOR ADDITION

A quantity can be a vector only if it obeys the laws of vector addition.

Following are the important properties of vector addition.

(*i*) Vectors of the same nature alone can be added. A force vector can be added to force vector only. It cannot be added to displacement vector.

(*ii*) Vector addition is commutative. *The sum of the vectors remains the same in whatever order they may be added.*

According to commutative law of vector addition,

$$\vec{a} + \vec{b} + \vec{c} + \ldots\ldots = \vec{b} + \vec{a} + \vec{c} + \ldots\ldots = \vec{c} + \vec{a} + \vec{b} + \ldots\ldots$$

The result of vector addition does not depend on the order in which the vector sum is written.

Proof. Let us prove the *commutative property of vector addition* in the case of two vectors $\vec{a}$ and $\vec{b}$.

Applying triangle law of vectors to the vector triangle ABC, we get

$$\vec{AC} = \vec{AB} + \vec{BC} \qquad \text{or} \qquad \vec{AC} = \vec{a} + \vec{b} \qquad \ldots(1)$$

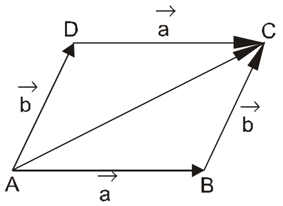


Fig. 2.1. Commutative property of vector addition

Again, applying triangle law of vectors to the vector triangle ADC, we get

$$\vec{AC} = \vec{AD} + \vec{DC} \quad \text{or} \quad \vec{AC} = \vec{b} + \vec{a} \qquad \ldots(2)$$

From (1) and (2), $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

(*iii*) Vector addition is distributive.

According to distributive law of vector addition,

$$\lambda(\vec{a} + \vec{b}) = \lambda\vec{a} + \lambda\vec{b}$$

(*iv*) Vector addition is associative. *The sum of the vectors remains the same in whatever grouping they are added.*

According to associative law of vector addition,

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

Proof. Let the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be represented by $\vec{PQ}$, $\vec{QS}$ and $\vec{ST}$ respectively. There are two ways to calculate the resultant of these vectors.

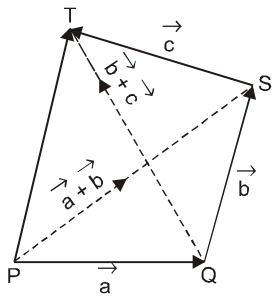


Fig. 2.2. Associative law of vector addition

The resultant of $\vec{a}$ and $\vec{b}$ is $\vec{PS}$ such that $\vec{PS} = \vec{a} + \vec{b}$

The resultant of $(\vec{a} + \vec{b})$ and $\vec{c}$ is $\vec{PT}$ such that

$$\vec{PT} = (\vec{a} + \vec{b}) + \vec{c} \qquad \ldots(1)$$

Again, if we add $\vec{b}$ and $\vec{c}$, we get $\vec{QT}$ such that $\vec{QT} = \vec{b} + \vec{c}$

The resultant of $\vec{a}$ and $(\vec{b} + \vec{c})$ is $\vec{PT}$ such that

$$\vec{PT} = \vec{a} + (\vec{b} + \vec{c}) \qquad \ldots(2)$$

From equations (1) and (2), $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ which proves the associative law of vector addition.

In terms of order and grouping, the rules for vector addition are the same as those of scalar addition.

2.2. RESOLUTION OF A VECTOR IN A PLANE

Resolution of a vector is the process of splitting the vector into two or more vectors in different directions which together produce the same effect as is produced by the given vector.

The vectors into which the given vector is splitted are called ***component vectors***.

Consider two non-zero vectors $\vec{a}$ and $\vec{b}$ in a plane [Fig. 2.3]. Let $\vec{A}$ be any other vector in this plane. Through the tail (P) of $\vec{A}$, draw a straight line parallel to $\vec{a}$.

Similarly, draw a straight line, parallel to $\vec{b}$, through the terminal point (Q) of $\vec{A}$. Let both the lines intersect at C.

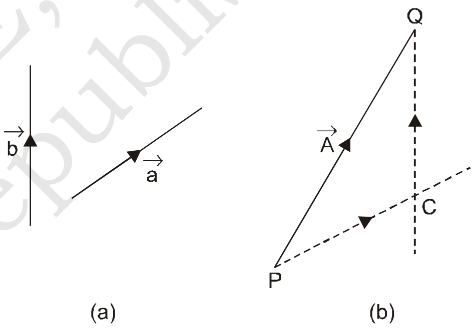


Fig. 2.3

Applying triangle law of vectors, $\vec{A} = \vec{PC} + \vec{CQ}$ ...(1)

As per the geometrical construction, $\vec{PC} = \lambda \vec{a}$ where λ is a real number. In the given case, λ is positive which indicates that $\vec{PC}$ is in the direction of $\vec{a}$. If λ were negative, then $\vec{PC}$ would have been opposite to $\vec{a}$.

Similarly, $\vec{CQ} = \mu \vec{b}$, where μ is another real number.

From equation (1), $\vec{A} = \lambda \vec{a} + \mu \vec{b}$

So, $\vec{A}$ has been resolved along $\vec{a}$ and $\vec{b}$.

It may be noted that $\vec{A}$ determines μ and λ unambiguously. The converse is also true, *i.e.,* each vector $\vec{A}$ in a plane is completely described or determined by a pair of real numbers λ, μ. The **uniqueness of the resolution procedure** is proved below.

Let us assume that there are two ways of resolving $\vec{A}$ along $\vec{a}$ and $\vec{b}$ such that

$$\vec{a} = \lambda\,\vec{a} + \mu\,\vec{b} = \lambda'\,\vec{a} + \mu'\,\vec{b}$$

$$\therefore \qquad (\lambda - \lambda')\,\vec{a} = (\mu' - \mu)\,\vec{b}$$

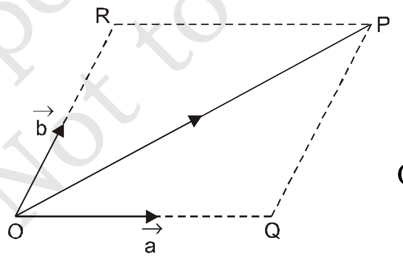
But $\vec{a}$ and $\vec{b}$ are different vectors.

So, the above equation is satisfied only if $\vec{a} = \vec{b} = \vec{0}$.

Thus, **there is one and only one way** in which a vector $\vec{A}$ can be resolved along $\vec{a}$ and $\vec{b}$. However, it may be pointed out here that a vector may be resolved into an infinite number of components. The reverse process, *i.e.,* the sum of the components will of course yield only the given vector.

In Fig. 2.4, the resolution of a position vector $\overrightarrow{OP}$ has been shown.

Applying parallelogram law of vectors, we can prove that $\lambda\,\vec{a}$ and $\mu\,\vec{b}$ are actually the components of $\overrightarrow{OP}$.



$$\overrightarrow{OQ} = \lambda\vec{a}, \quad \overrightarrow{OR} = \mu\vec{b}$$

Fig. 2.4. Resolution of vector

2.3. RECTANGULAR COMPONENTS

When a vector is splitted into two component vectors at right angles to each other, the component vectors are called the ***rectangular components*** *of the given vector.*

Consider a vector $\vec{A}$ represented by $\vec{OP}$. Through the point O, draw two mutually perpendicular axes—X-axis and Y-axis. Let the vector $\vec{A}$ make an angle θ with the X-axis. From the point P, drop a perpendicular PM on X-axis.

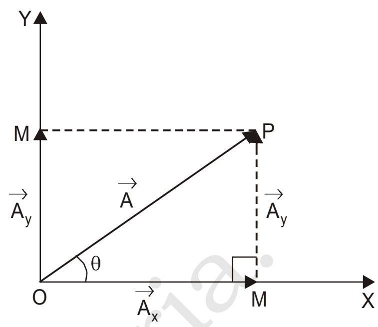


Fig. 2.5. Resolution of a vector into two rectangular components

Now $\vec{OM}\,(=\vec{A}_x)$ is the resolved part of $\vec{A}$ along X-axis. It is also known as the x-component of $\vec{A}$ or the horizontal component of $\vec{A}$. $\vec{A}_x$ may be regarded as the projection of $\vec{A}$ on X-axis.

$\vec{ON}$ (= $\vec{A}_y$) is the resolved part of $\vec{A}$ along Y-axis. It is also known as the y-component of $\vec{A}$ or the vertical component of $\vec{A}$. The vertical component of $\vec{A}$ may be regarded as the projection of $\vec{A}$ on Y-axis.

So, $\vec{A}_x$ and $\vec{A}_y$ are the rectangular components of $\vec{A}$.

Applying triangle law of vectors to the vector triangle OMP, we get

$$\vec{A}_x + \vec{A}_y = \vec{A}$$

This equation confirms that $\vec{A}_x$ and $\vec{A}_y$ are the components of $\vec{A}$.

In right-angled triangle OMP,

$$\cos\theta = \frac{A_x}{A} \quad \text{or} \quad A_x = A\cos\theta \qquad \ldots(1)$$

$$\sin\theta = \frac{A_y}{A} \quad \text{or} \quad A_y = A\sin\theta \qquad \ldots(2)$$

Squaring and adding (1) and (2), we get

$$A_x^2 + A_y^2 = A^2\cos^2\theta + A^2\sin^2\theta$$

or $$A_x^2 + A_y^2 = A^2(\cos^2\theta + \sin^2\theta)$$

$\therefore$ $$A_x^2 + A_y^2 = A^2 \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

or $$\boxed{A = \sqrt{A_x^{\,2} + A_y^{\,2}}}$$

This equation gives the magnitude of the given vector in terms of the magnitudes of the components of the given vector.

2.4. RESOLUTION OF A POSITION VECTOR INTO TWO RECTANGULAR COMPONENTS

Fig. 2.6 shows position vector $\vec{r}$ represented by $\overrightarrow{OP}$. Draw PM $\perp$ X-axis and PN $\perp$ Y-axis. $\overrightarrow{OM} = x\,\hat{i}$ and $\overrightarrow{ON} = y\,\hat{j}$.

According to parallelogram law of vector addition,

$$\overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{ON}$$

or $$\vec{r} = x\hat{i} + y\,\hat{j}$$

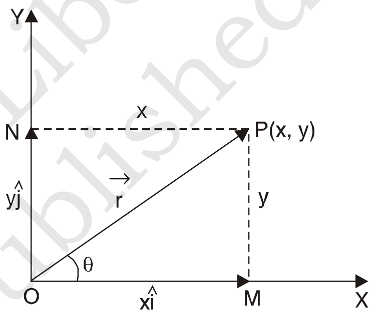


Fig. 2.6. Resolution of position vector

Let θ be the angle made by $\vec{r}$ with X-axis.

Then $x = r\cos\theta$ and $y = r\sin\theta$

$|\vec{r}|$ or $r = \sqrt{x^2 + y^2}$

2.5. EXAMPLE OF RESOLUTION OF VECTOR

An example of 'resolution of a vector' is 'walk of a man'. When a man walks, he presses the ground slantingly in the backward direction with a force F. The ground offers an equal reaction R in the opposite direction. The vertical component V of this reaction balances the weight of the man. The horizontal component H helps the man to walk.

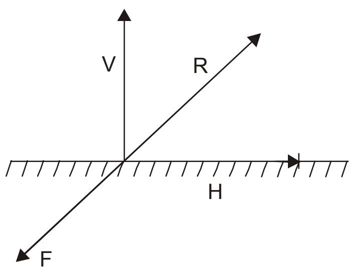


Fig. 2.7. Walk of a man

2.6. ADDITION OF VECTORS AND RECTANGULAR RESOLUTION

Consider two points P and Q having co-ordinates (x_1, y_1) and (x_2, y_2) respectively with reference to the origin O of the co-ordinate system. Let us first consider the position vector $\vec{r_1}$ which makes angle θ_1 with X-axis.

Now,
$$\vec{r_1} = \vec{x_1} + \vec{y_1} = x_1\hat{i} + y_1\hat{j}$$
$$x_1 = r_1 \cos \theta_1,\ y_1 = r_1 \sin \theta_1$$
$$r_1{}^2 = x_1{}^2 + y_1{}^2,\ \tan \theta_1 = \frac{y_1}{x_1}$$

Again, $(x_2 - x_1)$ and $(y_2 - y_1)$ are the components (in magnitude) of $\vec{PQ}$. Here, $y_2 - y_1$ is negative. [Note that $\vec{PQ}$ is directed from upper left to lower right.]

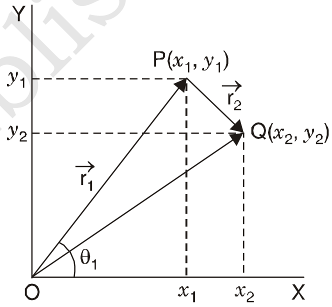


Fig. 2.8

Let us now add the components of $\vec{OP}$ to the components of $\vec{PQ}$.

Then, $x_1 + (x_2 - x_1) = x_2$, $y_1 + (y_2 - y_1) = y_2$

This gives us the components of $\vec{OQ}$.

So, we conclude that *the rule for addition of vectors can be broken down into two ordinary algebraic additions, one along each of the chosen axes. This directly implies that motion along a curve in a plane can be regarded as the sum of the independent linear motions, one along the X-axis and the other along Y-axis. The two linear motions may be treated separately and the results may be combined at the end.*

2.7. RESOLUTION OF A VECTOR INTO THREE RECTANGULAR COMPONENTS

Let a vector $\vec{A}$ be represented by $\vec{OP}$ as shown in Fig. 2.9. With O as origin, construct a rectangular parallelopiped with three edges along the three rectangular axes which meet at O. $\vec{A}$ becomes the diagonal of the parallelopiped. $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_y$ are three vector intercepts along x, y and z axes respectively. These are the three rectangular components of $\vec{A}$.

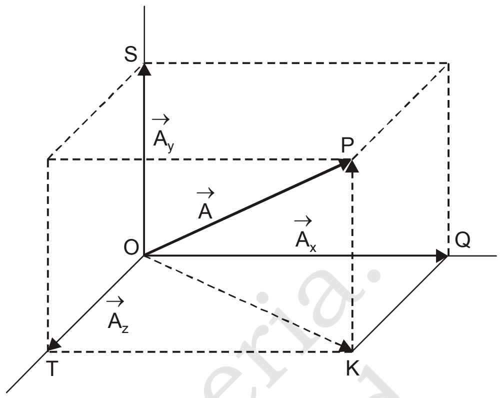


Fig. 2.9. Resolution of a vector into three rectangular components

Applying triangle law of vectors,

$$\vec{OP} = \vec{OK} + \vec{KP}$$

Applying parallelogram law of vectors, $\vec{OK} = \vec{OT} + \vec{OQ}$

$$\therefore \quad \vec{OP} = \vec{OT} + \vec{OQ} + \vec{KP}$$

But $$\vec{KP} = \vec{OS}$$

$$\therefore \quad \vec{OP} = \vec{OT} + \vec{OQ} + \vec{OS}$$

$$\vec{A} = \vec{A}_z + \vec{A}_x + \vec{A}_y$$

or $$\boxed{\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z}$$

or $$\boxed{\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}}$$

Again, $$OP^2 = OK^2 + KP^2$$

$$OP^2 = OQ^2 + QK^2 + KP^2$$

or $$OP^2 = OQ^2 + OT^2 + KP^2 \qquad [\because \quad QK = OT]$$

or $$A^2 = A_x^2 + A_z^2 + A_y^2 \qquad [\because \quad KP = OS = A_y]$$

or $$A^2 = A_x^2 + A_y^2 + A_z^2$$

or $$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$

This gives the magnitude of $\vec{A}$ in terms of the magnitudes of components $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$.

2.8. MOMENT OF FORCE (TORQUE)

(*a*) The rotational analogue of force is moment of force. It is also referred to as torque. This quantity measures the turning effect of a force.

The ***torque (or moment of force)*** *about an axis of rotation is a vector quantity, whose magnitude is equal to the product of magnitude of force and the perpendicular distance of the line of action of force from the axis of rotation and its direction is perpendicular to the plane containing the force and perpendicular distance.*

Fig. 2.10 shows a force $\vec{F}$ applied on a rigid body. The body is free to rotate about an axis passing through a point O and perpendicular to the plane of paper. If d is the perpendicular distance of the line of action of force from the point O, then the torque τ about the axis of rotation is : $\tau = Fd$.

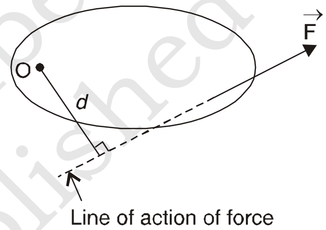


Fig. 2.10

The symbol τ stands for the Greek letter **tau.**

The torque is taken as positive if it tends to rotate the body anticlockwise. If the torque tends to rotate the body clockwise, then it is taken as negative.

The SI unit of torque is N m. Its dimensional formula is $[ML^2T^{-2}]$.

The dimensions of torque are the same as those of work or energy. It is, however, a very different physical quantity than work. Moment of force is a vector, while work is a scalar.

(*b*) Torque in Vector Notation. If a force $\vec{F}$ acts on a single particle at a point P whose position with respect to the origin O is given by the position vector $\vec{r}$, then the moment of force, acting on the particle, with respect to the origin O is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

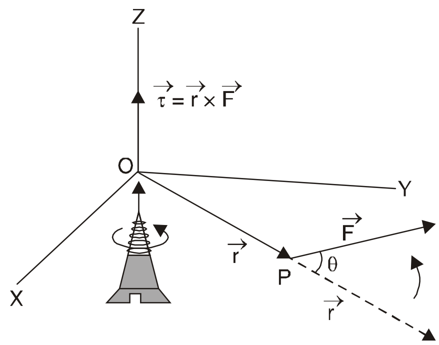


Fig. 2.11

The direction of $\vec{\tau}$ is perpendicular to the plane of $\vec{r}$ and $\vec{F}$. Its direction is given by right-handed screw rule or right-hand thumb rule.

The magnitude of $\vec{\tau}$ is given by $\tau = r\text{F} \sin \theta$ ...(1)

where r is the magnitude of the position vector $\vec{r}$ *i.e.,* the length OP, F is the magnitude of the force $\vec{\text{F}}$ and θ is the angle between $\vec{r}$ and $\vec{\text{F}}$.

Now, $\sin \theta = \dfrac{d}{r}$ or $d = r \sin \theta$

From $= n(1), \tau = \text{F}\,(r \sin \theta) = \text{F}r_{\perp} = \text{F}d$

Again $\tau = r\,(\text{F} \sin \theta) = r\,\text{F}_{\perp} = r\,\text{F}_{\theta}$

SPECIAL CASES

(*i*) If $r = 0$, then $\tau = 0$. Clearly, a force has no torque if it passes through the point O about which torque is to be calculated. *This explains as to why we cannot open or close a door by applying force at the hinges of the door.*

(*ii*) If $\theta = 0°$ or $180°$, then $\sin \theta = 0$.

$\therefore$ $t = r\,\text{F} \sin \theta =$ 0

In this case, the line of action of the force passes through point O. Thus, *if the line of action of force passes through point O, the torque is zero.*

(*iii*) If $\theta = 90°$, then $\sin \theta = \sin 90° =$ 1 (max. value). So, τ is maximum.

$$\tau_{\text{max.}} = r\text{F}$$

This explains as to why *a handle is fixed perpendicular to the plane of door.*

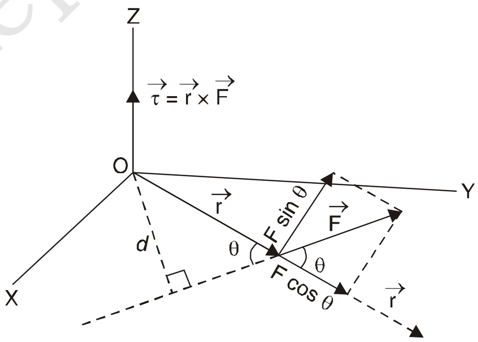


Fig. 2.12

2.9. COUPLE

A ***couple*** *is a set of two equal (in magnitude), opposite (in direction) forces having different lines of action.* A couple produces rotation without translation.

Properties of a Couple. (*a*) A couple produces or tends to produce only the rotational motion. (*b*) A couple cannot be replaced by a single force. (*c*) A couple can be shifted anywhere in its plane of action.

Torque is the turning effect produced by a single force. Couple is only a set of two equal (in magnitude), opposite (in direction) and parallel forces having different lines of action.

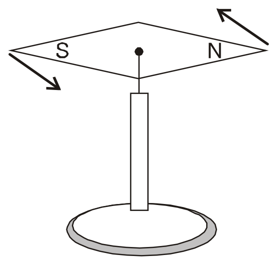


Fig. 2.13. The Earth's magnetic field exerts equal and opposite forces on the poles of a compass needle. These two forces form a couple

Moment of Couple. *It is the rotational effect produced by a couple.* It is a vector quantity. Its units and dimensions are the same as those of $\vec{\tau}$.

Expression for Moment of Couple. Let OX, OY and OZ be the three mutually perpendicular axes. Let two equal (in magnitude) and opposite (in direction) forces, $-\vec{F}$ and $\vec{F}$ act at P and Q respectively in the XOZ plane. The position vectors of P and Q with reference to origin O are given by $\vec{r_1}$ and $\vec{r_2}$ respectively.

Moment of force $-\vec{F}$ about O, $\vec{\tau_1} = \vec{r_1} \times (-\vec{F}) = -\vec{r_1} \times \vec{F}$

Moment of force $\vec{F}$ about O, $\vec{\tau_2} = \vec{r_2} \times \vec{F}$

Applying the right-hand rule for the cross product of vectors, we find that $\vec{\tau_1}$ acts along the negative direction of Y-axis and $\vec{\tau_2}$ acts along the positive direction of Y-axis as shown in Fig. 2.14.

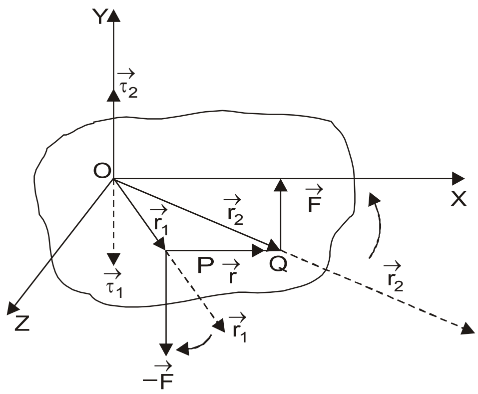


Fig. 2.14. Moment of couple

The moment of the couple $\vec{C}$ is *vector sum* of $\vec{r_1}$ and $\vec{r_2}$.

$$\therefore \qquad \vec{C} = \vec{\tau_1} + \vec{\tau_2}$$

$$= -\vec{r_1} \times \vec{F} + \vec{r_2} \times \vec{F} = \vec{r_2} \times \vec{F} - \vec{r_1} \times \vec{F}$$

or $$\vec{C} = (\vec{r_2} - \vec{r_1}) \times \vec{F}$$

Applying *triangle law of vectors* to the vector triangle OPQ, we get

$$\vec{r_1} + \vec{r} = \vec{r_2} \qquad \text{or} \qquad \vec{r_2} - \vec{r_1} = \vec{r}$$

$$\therefore \qquad \boxed{\vec{C} = \vec{r} \times \vec{F}}$$

The vector $\vec{r}$ lies in the plane of the two forces, *i.e.*, the plane XOZ. $\vec{C}$ is perpendicular to this plane.

Fig. 2.15. Our fingers apply a couple to turn the lid

2.10. WORK DONE IN ROTATING A RIGID BODY

Consider a rigid body which is capable of rotation about an axis through a point O of the rigid body and perpendicular to the plane of the paper.

Consider a point P such that the position vector of P with respect to O is $\vec{r}$ [Fig. 2.16]. Suppose an external force $\vec{F}$ is applied at the point P as shown. Let the body turn through an infinitesimally small angle $d\theta$ in a short time dt so that P moves to new position P′ such that $\vec{PP'} = \vec{ds}$.

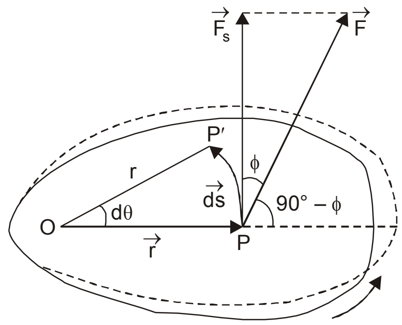


Fig. 2.16. Work done in rotational motion

*In magnitude, $ds = r\, d\theta$ ...(1)

Work dW done in rotating the body through a small angle $d\theta$ is given by

$$dW = \vec{F} \cdot \vec{ds} = F ds \cos \phi$$

---

* $\because \quad \theta = \frac{l}{r} \quad \therefore \quad l = r\theta$

Here, l is the length of an arc of a circle of radius r, θ is the angle subtended by the arc at the centre of circle.

where ϕ is the angle between $\vec{F}$ and $\vec{ds}$.

Now, $dW = (F \cos \phi)\, ds = F_s\, r\, d\theta$ [From (1)]

where F_s (= F cos ϕ) is the component of $\vec{F}$ in the direction of $\vec{ds}$. $\vec{F_s}$ is perpendicular to $\vec{r}$.

Again, $dW = (F \cos \phi)\, r\, d\theta = (r F \cos \phi)\, d\theta$

But $rF \cos \phi = rF \sin (90° - \phi) = |\vec{r} \times \vec{F}| = |\vec{\tau}| = \tau$

where $(90° - \phi)$ is the angle between $\vec{r}$ and $\vec{F}$. $\vec{\tau}$ is the moment of $\vec{F}$ about O.

$$\therefore \quad dW = \vec{\tau} \cdot \vec{d\theta}$$

Both $\vec{\tau}$ and $\vec{d\theta}$ act in the same direction. So, the angle between them is 0°.

$$\therefore \quad dW = \tau\, d\theta$$

$$\int dW = \int_0^\theta \tau\, d\theta \quad \text{or} \quad W = \tau \int_0^\theta d\theta = \tau(\theta - 0) = \tau\theta$$

or $$\boxed{W = \tau\, \theta}$$

Here it is assumed that τ is constant. Thus, the work done in rotating the body through a given angle is equal to the product of the torque and the angular displacement of the body.

2.11. POWER IN ROTATIONAL MOTION

Power, $$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta)$$

or $$P = \tau \frac{d}{dt}(\theta) \quad \text{or} \quad \boxed{P = \tau\, \omega}$$

Note that τ is being assumed as a constant.

2.12. EQUILIBRIUM OF RIGID BODIES

A **rigid body** is said to be in mechanical equilibrium if both its linear momentum and angular momentum are not changing with time, or equivalently the body has neither linear acceleration nor angular acceleration.

A rigid body such as a chair, a bridge or building is said to be in equilibrium if both the linear momentum and the angular momentum of the rigid body have a constant value. When a rigid body is in equilibrium, the linear acceleration of its centre of mass is zero. Also, the angular acceleration of the rigid body about any fixed axis in the reference frame is zero.

For the equilibrium of a rigid body, it is not necessary that the rigid body is at rest. However, *if the rigid body is at rest, then the equilibrium of the rigid body is called* ***static equilibrium.***

(*i*) First Condition for Equilibrium. The translational motion of the centre of mass of a rigid body is governed by the following equation:

$$\Sigma \vec{F}_{ext.} = \frac{d}{dt}(\vec{p})$$

Here $\Sigma \vec{F}_{ext.}$ is the vector sum of all the external forces that act on the rigid body.

For equilibrium, $\vec{p}$ must have a constant value.

$$\therefore \qquad \frac{d}{dt}(\vec{p}) = 0$$

$$\therefore \qquad \Sigma \vec{F}_{ext.} = 0$$

This vector equation is equivalent to three scalar equations:

$$\sum_{i=1}^{n} F_{ix} = 0,\ \sum_{i=1}^{n} F_{iy} = 0,\ \sum_{i=1}^{n} F_{iz} = 0 \qquad ...(1)$$

This leads us to ***the first condition for the equilibrium of rigid bodies.***

"The vector sum of all the external forces acting on the rigid body must be zero".

(*ii*) Second Condition for Equilibrium. The rotational motion of a rigid body is governed by the following equation:

$$\Sigma \vec{\tau}_{ext.} = \frac{d\vec{L}}{dt}$$

Here $\Sigma \vec{\tau}_{ext.}$ represents the vector sum of all the external torques that act on the body.

For equilibrium, $\vec{L}$ must have a constant value.

$$\therefore \qquad \frac{d}{dt}(\vec{L}) = 0$$

$$\therefore \qquad \sum \vec{\tau}_{ext.} = 0$$

This vector equation can be written as three scalar equations:

$$\sum_{i=1}^{n} \tau_{ix} = 0, \quad \sum_{i=1}^{n} \tau_{iy} = 0, \quad \sum_{i=1}^{n} \tau_{iz} = 0 \qquad \ldots(2)$$

This leads us to ***the second condition for the equilibrium of rigid bodies.***

"The vector sum of all the external torques acting on the rigid body must be zero."

The second condition for equilibrium is independent of the choice of the origin and the co-ordinate axes used for calculating the components of torques. If the net torque is zero, then its components are zero for any choice of *x, y* and *z* axes.

A body may be in partial equilibrium *i.e.,* it may be in translational equilibrium and not in rotational equilibrium or it may be in rotational equilibrium and not in translational equilibrium.

Consider a light (*i.e.,* of negligible mass) rod (AB), at the two ends (A and B) of which two parallel forces both equal in magnitude are applied perpendicular to the rod as shown in Fig. 2.17.

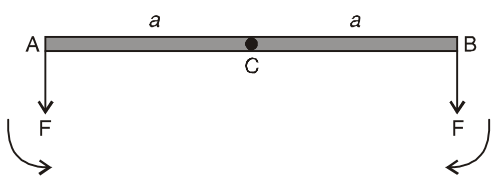


Fig. 2.17

Let C be the midpoint of AB. CA = CB = a. The moments of the forces at A and B, about C, will both be equal in magnitude (aF), but opposite in sense as shown. The net moment on the rod will be zero. The system will be in rotational equilibrium, but it will not be in translational equilibrium: $\Sigma F \neq 0$.

The force at B in Fig. 2.17 is reversed in Fig. 2.18. Thus, we have the same rod with two equal and opposite forces applied perpendicular to the rod, one at end A and the other at end B. Here the moments of both the forces are equal, but they are not opposite; they act in the same sense and cause anticlockwise rotation of the rod. The total force on the body is zero; so the body is in translational equilibrium; but it is not in rotational equilibrium. Although the rod is not fixed in any way, it undergoes pure rotation (*i.e.,* rotation without translation).

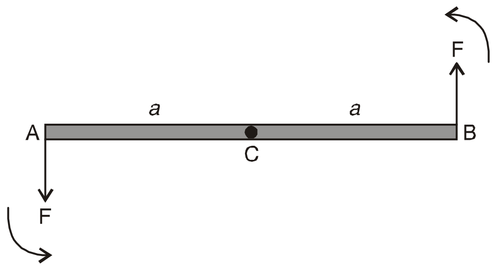


Fig. 2.18

2.13. PRINCIPLE OF MOMENTS (Case of Ideal Lever)

An ideal lever is essentially a light (*i.e.,* of negligible mass) rod pivoted at a point along its length. This point is called the fulcrum. A see-saw on the children's playground is a typical example of a lever. Two forces F_1 and F_2, parallel to each other and usually perpendicular to the lever, act on the lever at distances d_1 and d_2 respectively from the fulcrum as shown in Fig. 2.19.

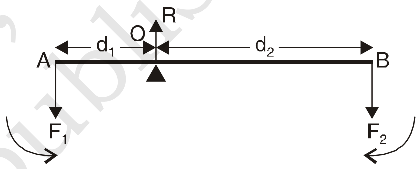


Fig. 2.19

Let R be the reaction of the support at the fulcrum. For translational equilibrium,

$$R - F_1 - F_2 = 0 \qquad ...(1)$$

For considering rotational equilibrium, we take the moments about the fulcrum ; the sum of moments must be zero.

$$F_1 d_1 - F_2 d_2 = 0 \qquad ...(2)$$

Normally the anticlockwise (clockwise) moments are taken to be positive (negative). Note R acts at the fulcrum itself and has zero moment about the fulcrum.

In the case of the lever, force F_1 is usually some weight to be lifted. It is called the *load* and its distance from the fulcrum d_1 is called the *load arm.* Force F_2 is the *effort* applied to lift the load ; distance d_2 of the effort from the fulcrum is the *effort arm.*

Eq. (2) can be written as

$$F_1 d_1 = F_2 d_2$$

or load × load arm = effort × effort arm

The above equation expresses the principle of moments for a lever. Incidentally the ratio F_1/F_2 is called the Mechanical Advantage (M.A.);

$$\text{M.A.} = \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

If the effort arm d_2 is larger than the load arm, the mechanical advantage is greater than one. Mechanical advantage greater than one means that a small effort can be used to lift a large load.

2.14. CENTRE OF GRAVITY

The centre of gravity of a body is a point where the weight of the body acts and total gravitational torque on the body is zero.

Consider an irregular-shaped cardboard and a narrow tipped object like a pencil. By trial and error, we can locate a point G on the cardboard where it can be balanced on the tip of the pencil. (The cardboard remains horizontal in this position.) This point of balance is the centre of gravity (CG) of the cardboard. The tip of the pencil provides a vertically upward force due to which the cardboard is in mechanical equilibrium. As shown in Fig. 2.20, the reaction of the tip is equal and opposite to $\overrightarrow{Mg}$, the total weight of (*i.e.,* the force of gravity on) the cardboard and hence the cardboard is in translational equilibrium. It is also in rotational equilibrium; if it were not so, due to the unbalanced torque it would tilt and fall. There are torques on the cardboard due to the forces of gravity like $\overrightarrow{m_1 g}$, $\overrightarrow{m_2 g}$ ...... etc., acting on the individual particles that make up the cardboard.

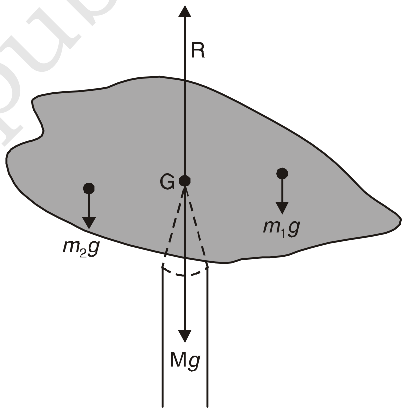


Fig. 2.20. Balancing a cardboard on the tip of a pencil. The point of support G is the centre of gravity

The CG of the cardboard is so located that the total torque on it due to the forces $\overrightarrow{m_1 g}$, $\overrightarrow{m_2 g}$ ...... etc. is zero.

If $\vec{r_i}$ is the position vector of the *i*th particle of an extended body with respect to its CG, then the torque about the CG, due to the force of gravity on the particle is $\vec{\tau_i} = \vec{r_i} \times \overrightarrow{m_i g}$. The total gravitational torque about the CG is zero, *i.e.*, $\vec{\tau_g} = \Sigma \vec{\tau_i} = \Sigma \vec{r_i} \times \overrightarrow{m_i g} = 0$ ...(1)

We may therefore, define the CG of a body as that point where the total gravitational torque on the body is zero.

In Eq. (1), $\vec{g}$ is the same for all particles, and hence it comes out of the summation. This gives, since $\vec{g}$ is non-zero,

$\Sigma m_i \vec{r_i} = 0$. The position vectors ($\vec{r_i}$) are taken with respect to the CG. So, the origin must be the centre of mass of the body. Thus, the centre of gravity of the body coincides with the centre of mass in uniform gravity or gravity-free space.

2.15. STABLE, UNSTABLE AND NEUTRAL EQUILIBRIUM OF RIGID BODIES

Something is in equilibrium when both the resultant force and resultant turning moment on it are zero.

Following are the three types of translational static equilibrium of a body.

(*i*) ***When potential energy is minimum, the particle is said to be in stable equilibrium.***

Any displacement of the particle from the equilibrium position will result in a restoring force. This restoring force shall try to return the particle to the equilibrium position.

If a body is in stable equilibrium, work must be done on it by an external agent to change its position. This results in an increase in its potential energy.

Consider a cube at rest on one face on a horizontal table. Fig. 2.21 shows the central cross-section of the cube. The centre of gravity is shown at the centre of this cross-section. Suppose a force F is applied to the cube so as to rotate it without slipping about an axis along an edge. The centre of gravity of the cube will be raised. Moreover, work is done on the cube. This increases the potential energy of the cube. If the force is removed, the cube tends to return to its original position. This initial position is clearly a stable equilibrium position.

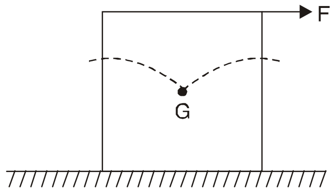


Fig. 2.21. Stable equilibrium

(*ii*) ***When the potential energy of a system is maximum, the system is in unstable equilibrium.***

Any displacement from 'unstable equilibrium position' will result in a force tending to push the system farther from the 'unstable equilibrium position'. No work is required to be done on the system by an external agent to change the position of the system. The displacement results in a decrease in the potential energy of the system.

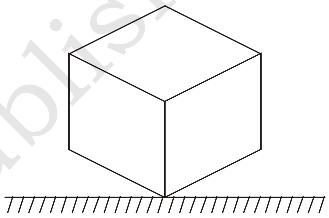


Fig. 2.22. Unstable equilibrium

A cube balanced on an edge can be considered in unstable equilibrium if a horizontal force is applied perpendicular to the edge. But the cube is in stable equilibrium with respect to a horizontal force parallel to the edge.

(*iii*) ***When the potential energy of a system is constant, the system is said to be in neutral equilibrium.***

When the system is displaced slightly, there is neither a repelling nor a restoring force.

A sphere (say, a football) on a horizontal table is a good illustration of neutral equilibrium. If a horizontal force is applied on the sphere, the centre of gravity of the sphere is neither raised nor lowered. The centre of gravity moves along the dashed line in Fig. 2.23. The potential energy of the sphere remains constant during displacement.

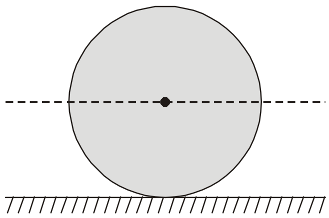


Fig. 2.23. Neutral equilibrium

2.16. CONCEPT OF CENTRE OF MASS

The centre of mass of a body is a point where the whole mass of the body is supposed to be concentrated for describing its translatory motion.

The centre of mass of a system of particles is that single point which moves in the same way in which a single particle having the total mass of the system and acted upon by the same external force would move.

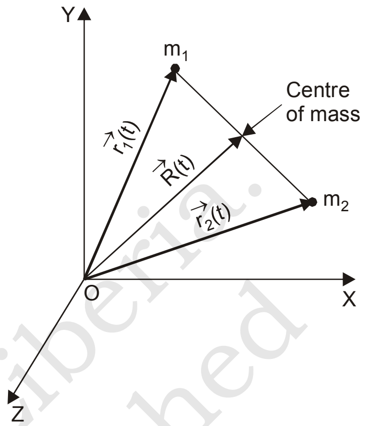


Fig. 2.24. Centre of mass of a two-particle system

The centre of mass of a system is only a point defined mathematically for the sake of convenience. It is not necessary that the total mass of the system be actually present at the centre of mass. As an example, centre of mass of a uniform circular ring is at the centre of the ring where there is no mass.

It may be noted that *it is not necessary that there may be a material particle at the centre of mass of the system.* But we can always calculate the position of the centre of mass at each time.

(*i*) For a two-particle system, the centre of mass always lies between the two particles and on the line joining them. In-fact $\vec{R}$ is a weighted average *i.e.,* each particle makes a contribution proportional to its mass.

(*ii*) When the two particles are of equal masses *i.e.*, $m_1 = m_2 = m$(say), then

$$\vec{R} = \frac{m\vec{r}_2 + m\vec{r}_2}{m + m} = \frac{\vec{r}_1 + \vec{r}_2}{2}$$

So, the centre of mass of two particles of equal masses lies exactly midway between them.

2.17. FRICTION

Friction *is the retarding force which is called into play when a body actually moves or tends to move over the surface of another body.*

Consider a block of mass m which is projected with initial velocity v along a long horizontal table. The block will finally come to rest. This means that while it is moving, it experiences an opposing force that points in a direction opposite to its motion. This opposing force is called force of friction.

Whenever the surface of one body slides over that of another, each body exerts a frictional force on the other. The frictional force on each body is in a direction opposite to its motion relative to the other body. *Frictional forces always oppose relative motion and never help it.* **Even when no relative motion is actually present but there is only a tendency for relative motion, frictional forces exist between surfaces.**

Friction is very important in our daily lives. Left to act alone, it brings every moving body to a stop. In an automobile, nearly 20% of the engine power is used to counteract frictional forces. Friction causes wear and tear of the moving parts and many engineering man-hours are devoted to reducing it. On the other hand, without friction, we could not walk, we could not hold a pen and if could it would not write; wheeled transport as we know it would not be possible.

Consider a block at rest on a horizontal table. We find that the block will not move even though we apply a small force [Fig. 2.25(1)]. The applied force is clearly balanced by an opposite frictional force exerted on the block by the table, acting along the surface of contact. As the applied force is gradually increased, the frictional force f_s also increases [Figs. 2.25(2) and 2.25(3)]. This indicates the **self-adjusting nature of the frictional force.**

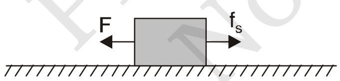


Fig. 2.25 (1)

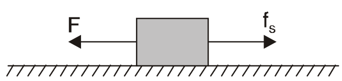


Fig. 2.25 (2)

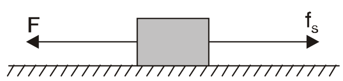


Fig. 2.25 (3)

*The frictional forces acting between surfaces at rest with respect to each other are called **forces of static friction.***

As we continue to increase the applied force, *we find some definite force at which the block just begins to move* [Fig. 2.25(4)]. At this stage, the maximum force of static friction acts. *The maximum force of static friction will be the same as the smallest force necessary to start motion.*

Once motion is started, the frictional force decreases so that a smaller force is necessary to maintain uniform motion [Fig. 2.25 (5)]. The forces acting between surfaces in relative motion are called forces of kinetic friction.

If the applied force is greater than the force of kinetic friction, then the block has accelerated motion [Fig. 2.25 (6)].

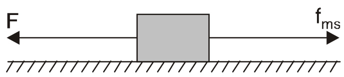


Fig. 2.25 (4)

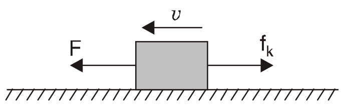


Fig. 2.25 (5)

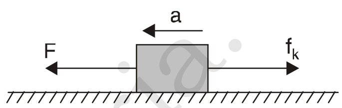


Fig. 2.25 (6)

2.18. STATIC FRICTION

It is the force of friction which exactly balances the applied force during the stationary state of the body. This frictional force exists when the bodies in contact are at rest with respect to each other. The force of static friction is a self-adjusting force *i.e.,* it adjusts its magnitude and direction so as to become exactly equal and opposite to the applied pull. The direction of the force of friction remains always opposite to the direction of the applied force.

Consider a block resting on a horizontal surface [Fig. 2.26]. Let a small pull P be applied on the body as shown. Let f_s be the resulting force of static friction. In the equilibrium position, the weight W of the body will be balanced by the normal reaction R. And the applied pull P will be balanced by the frictional force f_s.

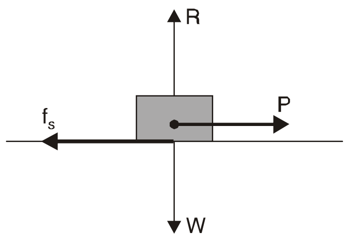


Fig. 2.26. Static friction

In vector notation, $\vec{W} = -\vec{R}$

and $\vec{P} = -\vec{f_s}$

2.19. LIMITING FRICTION

Limiting friction *is the maximum value of static friction which is called into play when a body is just going to start sliding over the surface of another body.*

When the applied pull P is increased, the static frictional force f_s also increases. However, there is a particular limit upto which the static frictional force f_s can increase. Beyond this limit, the applied pull P will be able to produce motion in the body.

2.20. LAWS OF LIMITING FRICTION

Following are the laws of limiting friction:

I. *The direction of the force of limiting friction is always opposite to that in which the motion tends to take place.*

II. *The limiting friction acts tangentially to the two surfaces in contact.*

III. *The magnitude of the limiting friction is directly proportional to the normal reaction between the two surfaces.*

IV. *The limiting friction depends upon the material and the nature of the surfaces in contact and their state of polish.*

V. *For any two given surfaces, the magnitude of the limiting friction is independent of the shape or the area of the surfaces in contact so long as the normal reaction remains the same.*

Experimental Verification. Consider a wooden block placed on a horizontal surface. It is attached to a string which passes over a frictionless pulley carrying a scale pan at the free end [Fig. 2.27]. Add weights in the scale pan till wooden block just starts sliding. It is evident that the force of friction was opposing the motion. This verifies law I.

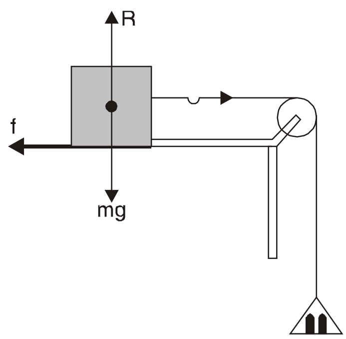


Fig. 2.27. Experimental verification of the laws of limiting friction

The force of friction f acts along the horizontal surface. This verifies law II.

When the block just starts sliding, the total weight added in the scale pan along with the weight of the scale pan is equal to the limiting friction. The normal reaction R is equal to the weight mg of the block. Now, put some known weight on the block. Determine the limiting friction again. It will be observed that the ratio of limiting friction and normal reaction is constant. In other words, the limiting friction is proportional to the normal reaction R. This verifies law III.

Let the wooden block be replaced by glass block of the same weight *mg*. It will be observed that the limiting friction will be different in this case. This verifies law IV.

If the wooden block is placed on its side instead of on its base, it will be observed that same force is required to move the block as when it was placed on its base. This verifies law V.

2.21. DYNAMIC OR KINETIC FRICTION

Dynamic *or* ***kinetic friction*** *comes into play if the two bodies in contact are in relative motion.* It acts in a direction opposite to the direction of the instantaneous velocity.

The dynamic or kinetic friction is of the following two types :

(*i*) Sliding friction. *It comes into play when a solid body slides over the surface of another body.*

(*ii*) Rolling friction. *It comes into play when a body rolls over the surface of another body.*

2.22. LAWS OF SLIDING FRICTION

(*i*) The sliding friction opposes the applied force and has a constant value, depending upon the nature of the two surfaces in relative motion.

(*ii*) The force of sliding friction is directly proportional to the normal reaction R.

(*iii*) The sliding frictional force is independent of the area of the contact between the two surfaces so long as the normal reaction remains the same.

(*iv*) The sliding friction does not depend upon the velocity, provided the velocity is neither too large nor too small.

2.23. VARIATION OF FRICTIONAL FORCE WITH THE APPLIED FORCE

It is illustrated graphically in Fig. 2.28. When there is no relative motion between the two bodies in contact, the frictional force increases at the same rate as the applied force.

If ON$'$ is the applied force, then ON is the frictional force such that

$$ON' = ON$$

The slope of the curve Oa is constant and is equal to unity.

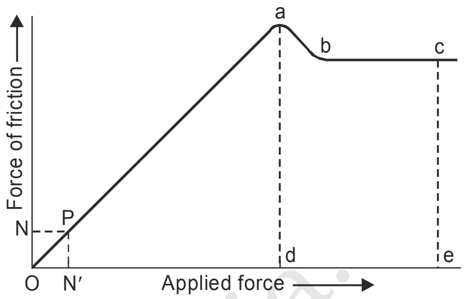


Fig. 2.28. Variation of frictional force with the applied force

When the applied force is equal to Od, the static frictional force becomes maximum. So, ad represents the limiting friction. When the applied pull exceeds the value Od, the body begins to move. At this stage, the frictional force suddenly decreases by a small amount and acquires a constant value *ce.* This value represents the dynamic or kinetic or sliding frictional force.

2.24. COEFFICIENT OF STATIC FRICTION

For any two surfaces in contact, it is the ratio of the limiting friction f_{ms} *and the normal reaction R between them.* It is denoted by μ_s.

$$\mu_s = \frac{f_{ms}}{R}$$

Since μ_s is a pure ratio therefore it has no units. The value of μ_s depends upon the state of polish of the two surfaces in contact. If the surfaces are smooth, the value of μ_s is small.

The force of static friction f_s is equal to the applied force. So, f_s can have any value from 0 to f_{ms}.

$\therefore \qquad f_s \leq f_{ms}$

[The equality sign holds only when f_s has its maximum value.]

$\therefore \qquad f_s \leq \mu_s R$

2.25. COEFFICIENT OF KINETIC FRICTION

It is defined as the ratio of kinetic friction and normal reaction. It is denoted by μ_k.

$\therefore \qquad \mu_k = \dfrac{f_k}{\text{R}}$

Now, $\qquad \dfrac{\mu_s}{\mu_k} = \dfrac{f_{ms}}{\text{R}} \times \dfrac{\text{R}}{f_k} = \dfrac{f_{ms}}{f_k}$

But $\qquad f_{ms} > f_k$

$\therefore \qquad \boxed{\mu_s > \mu_k}$

2.26. ANGLE OF FRICTION

It is the angle which the resultant of the force of limiting friction $\vec{f}_{ms}$ *and the normal reaction* $\vec{\text{R}}$ *makes with the normal reaction* $\vec{\text{R}}$.

Consider a block of weight $\vec{\text{W}}$ resting on a horizontal surface. The weight $\vec{\text{W}}$ will be balanced by the normal reaction $\vec{\text{R}}$ [Fig. 2.29].

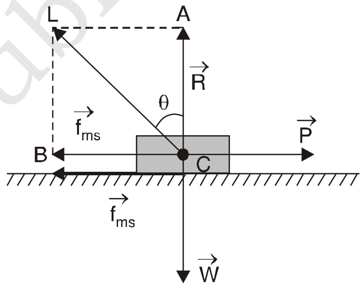


Fig. 2.29. Angle of friction

In vector notation, $\vec{\text{W}} = -\vec{\text{R}}$ (Newton's 3rd law of motion)

Now, apply a horizontal force $\vec{\text{P}}$ of such a magnitude that the block is about to move. Then, CB will represent the maximum force of static friction *i.e.,* limiting friction. The resultant of the limiting friction and the normal reaction is represented by the diagonal CL of the parallelogram CBLA. The angle θ which the resultant makes with the normal reaction is called the angle of friction.

In ΔCAL, $\qquad \tan\theta = \dfrac{\text{AL}}{\text{CA}} = \dfrac{\text{CB}}{\text{CA}} = \dfrac{f_{ms}}{\text{R}}$

But $\qquad \dfrac{f_{ms}}{\text{R}} = \mu_s \qquad$ (definition of coefficient of friction)

$\therefore \qquad \boxed{\tan\theta = \mu_s}$

So, *the tangent of the angle of friction is equal to the coefficient of static friction.*

2.27. ROLLING FRICTION

When a body rolls or tends to roll over the surface of another body, then both the rolling body and the surface on which it rolls are compressed by a small amount. As a result, the rolling body has to continuously climb a hill as shown [Fig. 2.30]. Apart from this, the rolling body has to continuously detach itself from the surface on which it rolls. This is opposed by the adhesive force between the two surfaces in contact. On account of both these factors, a force originates which retards the rolling motion. This retarding force is called the rolling friction. It is denoted by f_r.

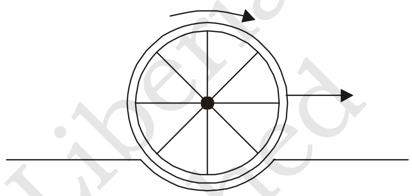


Fig. 2.30. Cause of rolling friction

Laws of rolling friction. The following laws of rolling friction are based on experiments.

(i) Rolling friction is directly proportional to normal reaction.

$$f_r \propto R$$

(ii) Rolling friction is inversely proportional to the radius of the rolling body.

$$f_r \propto \frac{1}{r}$$

Combining the two laws, we get

$$f_r \propto \frac{R}{r}$$

or

$$\boxed{f_r = \mu_r \times \frac{R}{r}} \qquad ...(1)$$

where μ_r is the coefficient of rolling friction, R is the normal reaction and r is the radius of the rolling body.

Comparison. For the same magnitude of normal reaction, the sliding friction is much greater than the rolling friction. That is why we prefer to convert sliding friction into rolling friction. The ball and roller bearings make use of this principle.

Illustration. The sliding friction of steel on steel is 100 times more than the rolling friction of steel on steel.

2.28. FRICTION IS A NECESSARY EVIL

Friction is a Necessity

(*i*) Without friction between our feet and the ground, it will not be possible to walk. When the ground becomes slippery after rain, it is made rough by spreading sand, etc.

(*ii*) The tyres of the vehicles are made rough to increase friction.

(*iii*) Various parts of a machine are able to rotate due to friction between belt and pulley.

Friction is an Evil

(*i*) Wear and tear of the machinery is due to friction.

(*ii*) Friction between different parts of the rotating machines produces heat and causes damage to them.

(*iii*) We have to apply extra power to machines in order to overcome friction. Thus, the efficiency of the machines decreases.

2.29. METHODS OF REDUCING FRICTION

(*i*) Polishing. The interlocking and the projections between the two surfaces are minimised and therefore the friction is reduced. This makes their life long.

(*ii*) Lubrication. A lubricant is a substance (a solid or a liquid) which forms thin layer between the two surfaces in contact. It fills the depressions present in the surfaces of contact and hence friction is reduced.

(*iii*) Streamlining. When a body moves past a fluid (liquid or air), the particles of the fluid move past it in regular lines of flow called streamlines. It is found that the resistance offered by the fluid to the body is minimum when its shape resembles that of streamlines. Thus the shape of automobiles is so designed that it resembles the streamline pattern and the resistance offered by the fluid is minimum.

(*iv*) Avoiding moisture. When the moisture is present, the friction is more. So, we must avoid moisture between the two surfaces.

(*v*) Use of alloys. Friction is reduced by lining the moving parts with alloys because alloys have low coefficients of friction.

(*vi*) Use of ball-bearings or roller-bearings. The rolling friction is much less than the sliding friction. So, we convert sliding friction into rolling friction. Even the axle is not allowed to move directly in the hub. The friction is further minimised by the use of roller-bearings or ball-bearings [Fig. 2.31].

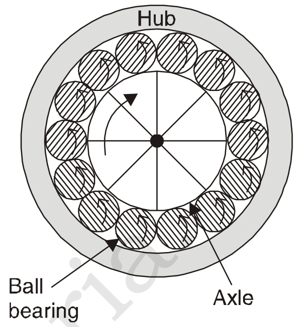


Fig. 2.31. Ball-bearings

2.30. WHY IS IT EASIER TO PULL A BODY THAN TO PUSH IT ?

Suppose a force P is applied to pull a block of weight W [Fig. 2.32]. The force P can be resolved into two rectangular components : P cos θ and P sin θ.

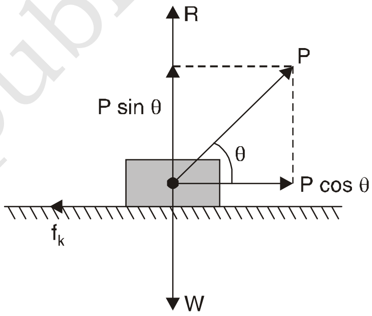


Fig. 2.32. Pulling a block

If R be the normal reaction, then

$$R = W - P \sin \theta$$

Force of kinetic friction,

$$f_k = \mu_k R$$

or $$f_k = \mu_k (W - P \sin \theta) \quad ...(1)$$

If a force P is applied to push a block of weight W [Fig. 2.33], then normal reaction,

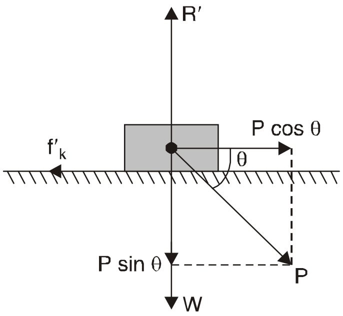


Fig. 2.33. Pushing a block

$$R' = W + P \sin \theta$$

force of kinetic friction, $f_k' = \mu_k R'$

or $$f_k' = \mu_k (W + P \sin \theta) \quad ...(2)$$

Comparing (1) and (2), we find that

$$f_k' > f_k$$

So, the frictional force is more in the case of push.

Hence, it is easier to pull than to push a body.

REVIEW EXERCISES

Do the review exercises in your notebook.

A. Multiple Choice Questions

1. A circular disk of radius R is made from an iron plate of thickness t and another disc Y of radius 4R is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moments of inertia I_X and I_Y is

(*a*) $I_Y = I_X$ (*b*) $I_Y = 64\ I_X$

(*c*) $I_Y = 32\ I_X$ (*d*) $I_Y = 16\ I_X$.

2. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Four objects, each of mass m, are kept gently on the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(*a*) $\frac{M\omega}{4m}$ (*b*) $\frac{M\omega}{4m}$

(*c*) $\frac{(M+4m)\,\omega}{M}$ (*d*) $\frac{(M-4m)\,\omega}{M+4m}$.

3. One end of a thin uniform rod of length L and mass M_1 is riveted to the centre of a uniform circular disc of radius r and mass M_2 so that both are coplanar. The centre of mass of the combination from the centre of the disc is (assume that the point of attachment is at the origin)

(*a*) $\frac{L(M_1+M_2)}{2M_1}$ (*b*) $\frac{LM_1}{2(M_1+M_2)}$

(*c*) $\frac{2(M_1+M_2)}{LM_1}$ (*d*) $\frac{2LM_1}{(M_1+M_2)}$.

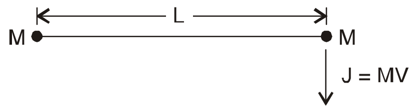
4. Two circular loops A and B of radii r_A and r_B respectively are made from the same uniform wire. The ratio of their moments of inertia about axes passing through their centres and perpendicular to their planes is $I_B/I_A = 8$. Then $(r_B/r_A) =$

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 8.

5. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse

J = MV is imparted to the body at one of its ends, what would be its angular velocity?



(*a*) V/L (*b*) 2V/L

(*c*) V/3L (*d*) V/4L.

6. A turntable rotates about a vertical axis with a constant angular speed ω. A circular pan rests on the turntable and rotates along with the table. The bottom of the pan is covered with a uniform thick layer of ice which also rotates with the pan. The ice starts melting. The angular speed of the turntable

(*a*) decreases (*b*) increases

(*c*) remains the same as ω (*d*) data insufficient.

7. Water is poured from a height of 10 m into an empty barrel at the rate of 1 litre per second. If the weight of the barrel is 10 kg, the weight indicated at time t = 60 s will be

(*a*) 71.4 kg (*b*) 68.6 kg

(*c*) 70.0 kg (*d*) 84.0 kg.

8. A force of 200 N is required to push a car of mass 500 kg slowly at constant speed on a level road. If a force of 500 N is applied, the acceleration of the car (in m s^{-2}) will be

(*a*) zero (*b*) 0.2

(*c*) 0.6 (*d*) 1.0.

9. When a bucket containing water is rotated fast in a vertical circle of radius R, the water in the bucket doesn't spill provided

(*a*) The bucket is whirled with a maximum speed of $\sqrt{2gR}$.

(*b*) The bucket is whirled around with a minimum speed of $\sqrt{\dfrac{gR}{2}}$.

(*c*) The bucket is having a r.p.m. of $n = \sqrt{\dfrac{900\,g}{\pi^2 R}}$.

(*d*) The bucket is having a r.p.m. of $n = \sqrt{\dfrac{3600\,g}{\pi^2 R}}$.

10. An insect is crawling up on the concave surface of a fixed hemispherical bowl of radius R. If the coefficient of friction is $\dfrac{1}{3}$, then the height up to which the insect can crawl is nearly

(*a*) 5% of R (*b*) 6% of R

(*c*) 6.5% of R (*d*) 7.5% of R.

B. Fill in the Blanks

1. A mass of 1 kg is just able to slide down the slope of an inclined rough surface when the angle of inclination is 60°. The minimum force necessary to pull the mass up the inclined plane is (g = 10 m s^{-2}) is __________ .
2. A block of mass m is resting on a smooth horizontal surface. One end of a uniform rope of mass (m/3) is fixed to the block, which is pulled in the horizontal direction by applying a force F at the other end. The tension in the middle of the rope is __________ .
3. A motor car is moving with a speed of 20 m s^{-1} on a circular track of radius 100 m. If its speed is increasing at the rate of 3 m s^{-1}, its resultant acceleration is __________ .
4. A body of mass 0.05 kg is observed to fall with an acceleration of 9.5 m s^{-2}. The opposing force of air on the body is __________ (g = 9.8 m s^{-2}).
5. A car of mass 1500 kg is moving with a speed of 12.5 m s^{-1} on a circular path of radius 20 m on a level road. The value of coefficient of friction between the tyres and road, so that the car does not slip, is __________ .

C. Very Short Answer Questions

1. Is it possible that a particle moving with constant speed may not have a constant velocity? If yes, give an example.
2. A stone is rotated in a circle with a string. The string suddenly breaks. In which direction will the stone move?
3. What is the source of centripetal force in the case of an electron revolving around the nucleus?
4. What is the effect on the direction of the centripetal force when the revolving body reverses its direction of motion?
5. Is it correct to say that the banking of roads reduces the wear and tear of the tyres of automobiles? If yes, explain.

D. Short Answer Questions

1. A stone tied to the end of a string is whirled in a horizotnal circle. When the string breaks, the stone flies away tangentially. Why?
2. What is the acceleration of a train travelling at 40 m s^{-1} as it goes round a curve of 160 m radius?
3. Is the angular velocity of rotation of hour hand of a watch greater or smaller than the angular velocity of Earth's rotation about its own axis?

4. (i) What is the direction of the angular velocity of the minute hand of a wall-clock?
   (ii) When the car takes a turn round a curve, a passenger sitting in the car tends to slide. To which side does the passenger slide?
   (iii) Comment on the statement 'sharper the curve, more is the bending'.
5. Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry?

E. Long Answer Questions

1. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.
2. Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $MR^2/4$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.
3. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?
4. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of the angular momentum of the cylinder about its axis?
5. A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rpm. How much is the angular speed of revolution of the child if he folds his hands back and thereby reduces his moment of inertia to $\frac{2}{5}$ times the initial value? Assume that the turntable rotates without friction.

SEMESTER-1 (Period-III)

TOPIC

3

Momentum and its Conservation

P11CH3

3.1. MOMENTUM (LINEAR)

Newton introduced the concept of momentum to measure the quantitative effect of force.

*The total quantity of motion possessed by a moving body is known as the **momentum of the body***. It is the product of the mass and velocity of a body. It is denoted by $\vec{p}$. $\vec{p} = m\vec{v}$

Since mass m is always positive therefore the direction of $\vec{p}$ is the same as that of $\vec{v}$.

In magnitude, $|\vec{p}| = m|\vec{v}|$ or $p = mv$

Since velocity is a vector and mass is a scalar therefore momentum is a vector. Again, $\vec{p}$ has same direction as that of $\vec{v}$ because m is always positive.

The cgs and SI units of momentum are g cm s^{-1} and kg m s^{-1} respectively.

The dimensional formula of momentum is [MLT $^{-1}$].

(i) When m is constant, $p \propto v$. This is shown in Fig. 3.1.

(ii) When v is constant, $p \propto m$. This is shown in Fig. 3.2.

(iii) When p is constant, then $v \propto \frac{1}{m}$. This is shown in Fig. 3.3.

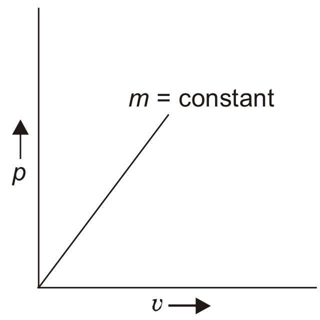


Fig. 3.1

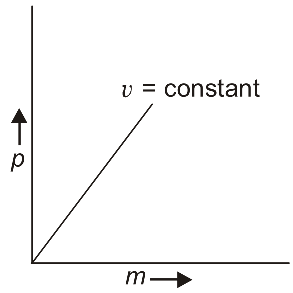


Fig. 3.2

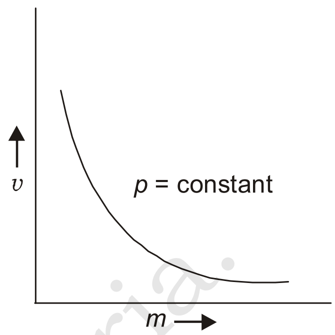


Fig. 3.3

Conceptual Problem 1. *A car and a scooter are travelling with the same speed. Which of the two has greater momentum?*

Ans. Let M and m be the masses of the car and scooter respectively. Let p_c and p_s be their respective momenta. Let v be the speed of both scooter and car.

Now, $$p_c = \mathrm{M}v \quad \text{and} \quad p_s = mv, \quad \frac{p_c}{p_s} = \frac{\mathrm{M}v}{mv} = \frac{\mathrm{M}}{m}$$

$\because \quad \mathrm{M} > m \qquad \therefore \quad p_c > p_s$

So, the momentum of the car is greater than the momentum of the scooter.

Conceptual Problem 2. *A car and a scooter have the same momentum. Which of the two has greater speed?*

Ans. In this case, $p = \mathrm{M}v_c = mv_s$

where v_c and v_s are the speeds of the car and scooter respectively.

Now, $$\frac{v_c}{v_s} = \frac{\mathrm{M}}{m}. \quad \because \quad m < \mathrm{M} \qquad \therefore \quad v_c < v_s$$

So, the speed of the car is less than the speed of the scooter.

Conceptual Problem 3. *Establish a general relation between momentum p and kinetic energy E_k.*

Ans. $$p = mv\ ;\ p^2 = m^2v^2$$

or $$p^2 = 2\ m\ \frac{1}{2}mv^2 = 2m\,\mathrm{E}_k \quad \text{or} \quad p = \sqrt{2m\,\mathrm{E}_k}$$

3.2. NEWTON'S SECOND LAW OF MOTION

(*i*) Statement. *The time rate of change of momentum of a body is directly proportional to the impressed force and takes place in the direction of the force.*

(*ii*) Explanation of Newton's second law. The statement can be divided into the following two parts:

(*a*) *The time rate of change of momentum of a body is proportional to the impressed force.*

A force acting on a body produces a certain change in the momentum of the body. When the given force is doubled, the 'change in momentum' of the body is also doubled. So, as the applied force is increased, the rate of change of momentum of the body is also increased.

(*b*) *The change of momentum takes place in the direction of the force.*

Consider a body to be at rest. When a force is applied on this body, the body will begin to move in the direction of the force. If a force is applied on a moving body in the direction of motion of the body, then there is an increase in the momentum of the body. However, if the force is applied on a moving body in a direction opposite to the direction of motion of the body, then there is a decrease in the momentum of the body.

(*iii*) Formula for force. Let a constant external force $\vec{F}$ acting on a body change its momentum from $\vec{p}$ to $\vec{p} + d\vec{p}$ in time interval dt. Then, the time rate of change of linear momentum is $\frac{d\vec{p}}{dt}$.

According to Newton's second law of motion,

$$\frac{d\vec{p}}{dt} \propto \vec{F} \quad \text{or} \quad \vec{F} \propto \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F} = k\frac{d\vec{p}}{dt}$$

Here k is a constant of proportionality. The value of k depends upon the units selected for the measurement of force. In both SI and cgs system, the unit of force is so chosen that $k = 1$.

$$\therefore \qquad \vec{F} = \frac{d\vec{p}}{dt}$$

3.3. IMPULSE

The effectiveness of a force in producing motion depends not only upon the magnitude of the force but also on the time for which the force acts. When a large force acts for an extremely short duration, neither the magnitude of the force nor the time for which it acts is important. In such a case, the total effect of force is measured. **The total effect of force is called impulse.** It may also be defined as a measure of the action of force. It is a vector quantity and is denoted by $\vec{J}$. It is the product of force and the time for which the force acts.

Suppose a force $\vec{F}$ acts for a short time dt. The impulse of this force is given by, $d\vec{J} = \vec{F}\,dt$

If we consider a finite interval of time from t_1 to t_2, then the impulse is given by,

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}\,dt$$

The right hand side of the above equation represents the impulse of varying force.

$$\vec{J} = \vec{F}\int_{t_1}^{t_2} dt = \vec{F}\Big[t\Big]_{t_1}^{t_2} = \vec{F}(t_2 - t_1)$$

or $$\vec{J} = \vec{F}\Delta t \qquad \text{where } \Delta t = t_2 - t_1$$

So, the impulse of a constant force $\vec{F}$ is equal to the product of the force and time interval Δt for which the force acts.

The direction of $\vec{J}$ is the same as the direction of $\vec{F}$.

3.4. UNITS AND DIMENSIONS OF IMPULSE

In **cgs system,** the unit of impulse is dyne second or g cm s^{-1}.

In **SI**, it is measured in newton second or kg m s^{-1}.

The dimensional formula of impulse is $[MLT^{-1}]$.

3.5. IMPULSE-MOMENTUM THEOREM

Impulse is measured by the total change in momentum that the force produces in a given time.

According to Newton's second law of motion, $\vec{F} = \frac{d\vec{p}}{dt}$

where $\vec{p}$ is the momentum of body at any time t and $\vec{F}$ is the applied force at that time.

$$d\vec{p} = \vec{F}\,dt$$

Integrating, $$\int_{p_1}^{p_2} d\vec{p} = \int_0^t \vec{F}\,dt$$

where $\vec{p_1}$ is the momentum at $t = 0$ and $\vec{p_2}$ is the momentum at time t.

$$\left[\vec{p}\right]_{p_1}^{p_2} = \int_0^t \vec{F}\,dt \quad \text{or} \quad \vec{p_2} - \vec{p_1} = \int_0^t \vec{F}\,dt$$

or $$\int_0^t \vec{F}dt = \vec{p_2} - \vec{p_1} \qquad \ldots(1)$$

So, *the impulse of a varying force is equal to the change in momentum produced by the force.*

If the applied force $\vec{F}$ is constant, then from equation (1),

$$\vec{F}\int_0^t dt = \vec{p_2} - \vec{p_1} \quad \text{or} \quad \vec{F}\left[t\right]_0^t = \vec{p_2} - \vec{p_1}$$

or $$\vec{F}(t-0) = \vec{p_2} - \vec{p_1} \quad \text{or} \quad \vec{F}\,t = \vec{p_2} - \vec{p_1}$$

Thus, *the impulse of a constant force is equal to the change of momentum.*

In the case of positive impulse acting on a body, there is an algebraic increase in the momentum of the body. If the impulse is zero, then there is no change in the momentum. In the case of negative impulse, there is a decrease in the momentum.

3.6. PRACTICAL APPLICATIONS OF IMPULSE

These are based on the fact that if the total change in momentum takes place in a very short time, then the force is large. If the change in momentum takes place over a longer interval of time, then the force is small.

If two forces $\vec{F}_1$ and $\vec{F}_2$ act on a body to produce the same impulse, then their respective times of application t_1 and t_2 should be such that

$$\vec{F}_1 t_1 = \vec{F}_2 t_2$$

Following are the *practical applications* of impulse.

1. *While catching a fast moving cricket ball, a player lowers his hands.* In this way, the time of catch increases and the force decreases. So, the player has to apply a less average force. Consequently, the ball will also apply only a small force (reaction) on the hands. In this way, the player will not hurt his hands.

2. *Automobiles are provided with spring systems.* When the automobile bumps over an uneven road, it receives a jerk. The spring increases the time of the jerk, thereby reducing the force. This minimises the damage to the automobile. [For the same reason, buffers are provided between the bogies of a train.]

3. *China plates are wrapped in paper or straw pieces while packing.* If, during transportation, the package gets a jerk, the time of blow will be increased. This will reduce the force of blow. In this way, the china plates will be saved from damage.

4. *It is difficult to catch a cricket ball as compared to a tennis ball moving with the same velocity.* This is due to the fact that the cricket ball is heavier than a tennis ball. The change in momentum is more in the case of a cricket ball than in the case of a tennis ball. As a result, more force is required to be applied in the case of a cricket ball.

5. *When a moving vehicle strikes against a wall, a large amount of force acts on the vehicle.* This is because the change in momentum is very large and is brought about in a very short interval of time. So, a large amount of force acts on the vehicle and the vehicle is damaged.

Example 1. *Force-time graph for a body is shown in Fig. 3.4. What is the velocity of the body at the end of 11 second ? Mass of the body is 7 kg. Assume the body to be starting from rest.*

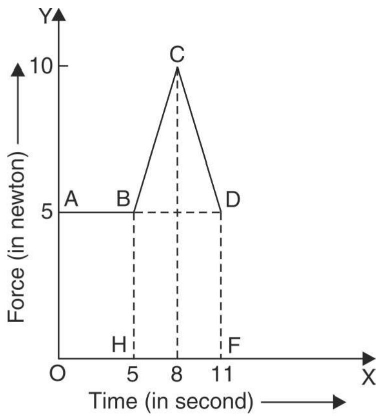


Fig. 3.4

Solution. Area ABHO = 5 × 5 = 25 units

Area BDFH = 5(11 – 5) = 30 units

Area BCD = $\frac{1}{2}$ × 6 × 5 = 15 units

Total area under the curve = (25 + 30 + 15) units = 70 units

Since the area under F-*t* curve gives impulse *i.e.*, change in momentum,

$$\therefore \qquad mv - 0 = 70 \quad \text{or} \quad v = \frac{70}{m} = \frac{70}{7}\ \text{m s}^{-1} = \mathbf{10\ m\ s^{-1}}$$

Example 2. *A ball moving with a momentum of 5 kg m s^{-1} strikes against a wall at an angle of 45° and is reflected at the same angle. Calculate the change in momentum (in magnitude).*

Solution. Let $\vec{p}_1$ and $\vec{p}_2$ be the initial and final momenta respectively of the ball.

Change in momentum = $\vec{p}_2 - \vec{p}_1$

$= \vec{p}_2 + (-\vec{p}_1) = \vec{AB}$

From the Fig. 3.5,

$$AB = \sqrt{p_1^2 + p_2^2}$$

$$= \sqrt{5^2 + 5^2}\ \text{kg m s}^{-1}$$

or $\quad AB = \sqrt{50}\ \text{kg m s}^{-1} = \mathbf{7.07\ kg\ m\ s^{-1}}$

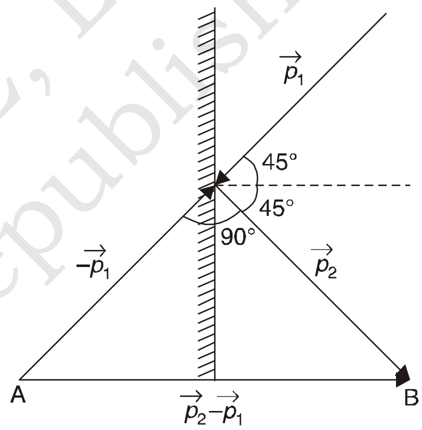


Fig. 3.5

3.7. NEWTON'S THIRD LAW OF MOTION

Forces acting on a body originate in other bodies that make up its environment. This property of forces was first stated by Newton in his third law of motion:

"To every action, there is always an equal (in magnitude) and opposite (in direction) reaction."

This law may also be stated as under:

"Action and reaction are equal in magnitude, opposite in direction and act on different bodies."

Consider interaction (action and reaction) between two bodies A and B. Let $\vec{F}_{BA}$ be the force exerted by A on B and $\vec{F}_{AB}$ the force exerted by B on A (Fig. 3.6). Then, according to Newton's third law of motion,

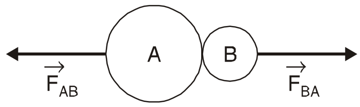


Fig. 3.6. Newton's third law of motion

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

It is clear from this equation that the two forces are equal in magnitude but opposite in direction. These forces of action and reaction act along the line joining the centres of two bodies.

One of the two forces involved in the interaction between two bodies may be called 'action' force. The other force will be called the 'reaction' force. The forces of action and reaction constitute a mutual simultaneous interaction. It cannot be said that action is the cause of reaction or reaction is the effect of action.

Newton's third law of motion leads us to a very interesting fact about forces. It is that the forces always exist in pairs. They never exist singly.

3.8. ELASTIC COLLISIONS AND ELEMENTARY IDEA OF INELASTIC COLLISIONS

A ***collision*** *is said to take place when either two bodies physically collide against each other or when the path of one body is changed by the influence of the other body.*

As a result of collision, the momentum and kinetic energy of the interacting bodies change. The forces involved in a collision are action-reaction forces, *i.e.,* the internal forces of the system. So, the total momentum is conserved. Also, the total energy is conserved.

Elastic Collision. *A collision is said to be an elastic collision if both the kinetic energy and momentum are conserved in the collision.*

During collision, the bodies are deformed. However, they regain their original shape completely if the collision is elastic. The mechanical energy is not converted into any other form of energy. In an elastic collision, the forces of interaction are conservative in nature.

Inelastic Collision. *A collision is said to be an inelastic collision if the kinetic energy is not conserved in the collision. However the momentum is conserved.*

The kinetic energy lost in the collision appears in the form of heat energy, sound energy, light energy, etc. The forces of interaction in an inelastic collision are non-conservative in nature.

If a ball is dropped from a certain height and the ball is unable to rise completely to its original height, then it would mean that ball has lost some kinetic energy (which would appear as heat energy). This would mean that collision is an inelastic collision.

Characteristics of Inelastic Collisions

(i) Kinetic energy is not conserved. ***(ii)*** Total energy is conserved. ***(iii)*** Momentum is conserved. ***(iv)*** Some or all of the forces involved in the collision are non-conservative. ***(v)*** A part of the mechanical energy is converted into heat, light, sound, etc.

3.9. HEAD-ON ELASTIC COLLISION [ONE-DIMENSIONAL ELASTIC COLLISION]

One-dimensional elastic collision *is that elastic collision in which the colliding bodies move along the same straight line path before and after the collision.*

Consider two bodies A and B of masses m_1 and m_2 respectively moving along the same straight line in the same direction [Fig. 3.7]. Let $\vec{v}_{1i}$ and $\vec{v}_{2i}$ be their respective velocities such that $|\vec{v}_{1i}| > |\vec{v}_{2i}|$.

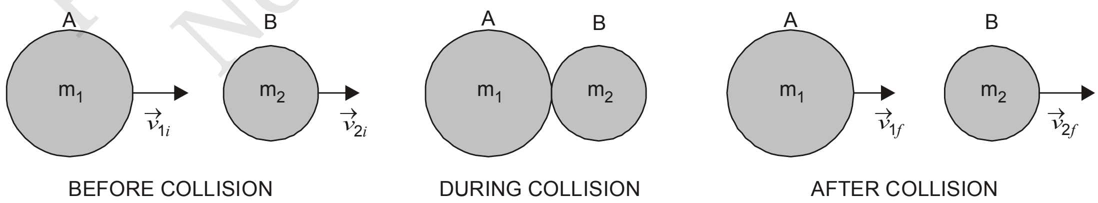


Fig. 3.7. One-dimensional elastic collision

The two bodies will collide after some time.

During collision, the bodies will be deformed in the region of contact. So, a part of the kinetic energy will be converted into potential energy.

The bodies will regain their original shape due to elasticity. The potential energy will be reconverted into kinetic energy. The bodies will separate and continue to move along the same straight line in the same direction but with different velocities.

- In an elastic collision, the kinetic energy onservation does not hold at every instant of ollision. It holds after the collision is over.
- Total linear momentum is conserved both in lastic and inelastic collisions.
- Total linear momentum is conserved at each instant of elastic and inelastic collisions.
- Total energy is conserved in all collisions.

Let *$\vec{v}_{1f}$ and $\vec{v}_{2f}$ be the velocities of A and B respectively after the collision.

Applying the law of conservation of momentum,

total momentum before collision = total momentum after collision

$\therefore$ $$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \text{ (in magnitude)}$$

or $$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i}) \quad ...(1)$$

Since the collision is elastic therefore kinetic energy will be conserved.

$\therefore$ Kinetic energy before collision = Kinetic energy after collision

$\therefore$ $$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

or $$m_1 v_{1i}^2 + m_2 v_{2i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

or $$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2)$$

or $$m_1 (v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2 (v_{2f} + v_{2i})(v_{2f} - v_{2i}) \quad ...(2)$$

Dividing (2) by (1), we get $v_{1i} + v_{1f} = v_{2f} + v_{2i}$

or $$v_{1i} - v_{2i} = v_{2f} - v_{1f} \quad ...(3)$$

*The velocity $\vec{v}_{2f}$ has to be greater than velocity $\vec{v}_{1f}$ because otherwise the two colliding bodies cannot separate.

*$(v_{1i} - v_{2i})$ is the magnitude of the relative velocity of A w.r.t. B. **$(v_{2f} - v_{1f})$ is the magnitude of relative velocity of B w.r.t. A. It may be noted that the direction of relative velocity is reversed after the collision.

Relative velocity of A w.r.t. B before collision

= Relative velocity of B w.r.t. A after collision

or Relative velocity of approach = Relative velocity of separation

In one-dimensional elastic collision, the relative velocity of approach before collision is equal to the relative velocity of separation after the collision.

From equation (3), $v_{2f} = v_{1i} - v_{2i} + v_{1f}$

From equation (1), $m_1(v_{1i} - v_{1f}) = m_2(v_{1i} - v_{2i} + v_{1f} - v_{2i})$

or $$m_1v_{1i} - m_1v_{1f} = m_2v_{1i} - 2m_2v_{1f} + m_2v_{1f}$$

or $$-m_1v_{1f} - m_2v_{1f} = -m_1v_{1i} + m_2v_{1i} - 2m_2v_{2i}$$

or $$(m_1 + m_2)\,v_{1f} = (m_1 - m_2)\,v_{1i} + 2m_2v_{2i}$$

or $$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} \qquad ...(4)$$

Again, from equation (3), $v_{1f} = v_{2f} - v_{1i} + v_{2i}$

Substituting this value in equation (1) and simplifying, we get

$$v_{2f} = \frac{m_2 - m_1}{m_1 + m_2}v_{2i} + \frac{2m_1}{m_1 + m_2}v_{1i} \qquad ...(5)$$

Equations (4) and (5) give the final velocities of the colliding bodies in terms of their initial velocities.

3.10. APPLICATION OF ELASTIC COLLISIONS

In a nuclear reactor, the neutrons are produced from the fission of Uranium. These neutrons are very fast. So, they cannot be used to produce more fission. Thus, they have to be quickly slowed down. This is done by making them collide against a target. If the targets are electrons, then the speed of neutrons will remain practically unchanged.

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* $(\vec{v}_{1i} - \vec{v}_{2i})$ is the relative velocity of approach.

** $(\vec{v}_{2f} - \vec{v}_{1f})$ is the relative velocity of separation.

This is because neutrons are massive as compared to electrons. If the targets are lead nuclei, then the neutrons merely bounce back with nearly the same speed. This is because neutrons are much lighter than lead nuclei.

If the targets are protons, then the neutrons are sufficiently slowed down because the masses of two colliding particles are comparable.

The protons are available in water. So, water can be used as a moderator in a nuclear reactor. But neutrons tend to constitute stable nuclei with protons. So, instead of water, we use heavy water (D_2O) as moderator. The nucleus of deuterium contains one neutron and one proton only.

Example 3. *Two bodies of masses 50 g and 30 g moving in the same direction, along the same straight line with velocities 50 cm s^{-1} and 30 cm s^{-1} respectively suffer one-dimensional elastic collision. Calculate their velocities after the collision.*

Solution. Mass, m_1 = 50 g ; Mass, m_2 = 30 g ;

Velocity, v_{1i} = 50 cm s^{-1} ; Velocity, v_{2i} = 30 cm s^{-1}

v_{1f} = ?, v_{2f} = ?

$${}_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$= \left(\frac{50-30}{50+30} \times 50 + \frac{2 \times 30}{50+30} \times 30\right) \text{cm s}^{-1} = \mathbf{35 \text{ cm s}^{-1}}$$

Again,

$$v_{2f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2i} + \frac{2m_1}{m_1 + m_2} v_{1i}$$

$$= \left(\frac{30-50}{50+30} \times 30 + \frac{2 \times 50}{50+30} \times 50\right) \text{cm s}^{-1} = \mathbf{55 \text{ cm s}^{-1}}$$

Example 4. *A body A of mass 2 kg moving with a velocity of 25 m s^{-1} in the east direction collides elastically with another body B of mass 3 kg moving with velocity of 15 m s^{-1} westwards. Calculate the velocity of each ball after the collision.*

Solution. m_1 = 2 kg, v_{1i} = 25 m s^{-1}, m_2 = 3 kg ;

*v_{2i} = – 15 m s^{-1}, v_{1f} = ?, v_{2f} = ?

* The initial velocity of the body B is in a direction opposite to that of the initial velocity of body A.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$= \left(\frac{2-3}{2+3} \times 25 + \frac{2 \times 3}{2+3} \times -15 \right) \text{m s}^{-1} = \mathbf{-\,23\ m\ s^{-1}}$$

$$v_{2f} = \frac{m_2 - m_1}{m_1 + m_2} v_{2i} + \frac{2m_1}{m_1 + m_2} v_{1i}$$

$$= \left(\frac{3-2}{3+2} \times -15 + \frac{2 \times 2}{3+2} \times 25 \right) \text{m s}^{-1} = \mathbf{17\ m\ s^{-1}}$$

3.11. INELASTIC COLLISION AND COEFFICIENT OF RESTITUTION

The ratio of relative speed of separation after collision and the relative speed of approach before collision is a constant. This constant is called ***coefficient of restitution*** or ***coefficient of resilience.*** It is denoted by *e*. It is a measure of the degree of elasticity of a collision. Its value depends upon the nature of the colliding bodies.

The coefficient of restitution is defined as the ratio of the magnitude of relative velocity of separation after collision to the magnitude of relative velocity of approach before collision.

$$e = \frac{|\vec{v}_{2f} - \vec{v}_{1f}|}{|\vec{v}_{1i} - \vec{v}_{2i}|}$$

(i) In a perfectly elastic collision, the relative velocity of separation is equal to the relative velocity of approach.

$\therefore$ $e = 1$

Note that there is no loss of kinetic energy. A body dropped from a certain height shall rebound to the same height.

(ii) In a perfectly inelastic collision, the bodies stick together after the collision. The relative velocity of separation is zero.

$\therefore$ $e = 0$

(iii) In general, the bodies are neither perfectly elastic nor perfectly inelastic. In that case, velocity of separation = e(velocity of approach), where $0 < e < 1$.

For two lead balls, $e = 0.20$ and for the glass balls, $e = 0.95$.

(iv) If $e > 1$, then the collision is superelastic collision. [An example of superelastic collision is that of a cracker which is forcefully struck against the ground.]

3.12. ONE-DIMENSIONAL INELASTIC COLLISION

A collision is said to be one-dimensional inelastic collision if the momentum is conserved with some loss of kinetic energy and the colliding bodies continue to move along the same straight line path before and after the collision.

Consider two bodies A and B of masses m_1 and m_2 moving, in the same direction, along the same straight line path with velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$ respectively such that $|\vec{v}_{1i}| > |\vec{v}_{2i}|$. The two bodies A and B undergo head-on collision. After the collision, they continue to move along the same straight line with velocities $\vec{v}_{1f}$ and $\vec{v}_{2f}$ respectively without any change in direction.

Using conservation of momentum,

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad \ldots(1)$$

If e be the coefficient of restitution, then

$$e = \frac{v_{2f} - v_{1f}}{v_{1i} - v_{2i}}$$

or

$$v_{2f} = v_{1f} + e(v_{1i} - v_{2i}) \qquad \ldots(2)$$

Substituting the value of v_{2f} in equation (1),

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2[v_{1f} + e(v_{1i} - v_{2i})]$$

or

$$(m_1 + m_2)\, v_{1f} = (m_1 - em_2)\, v_{1i} + (1 + e)\, m_2 v_{2i}$$

or

$$v_{1f} = \frac{m_1 - em_2}{m_1 + m_2} v_{1i} + \frac{(1+e)m_2}{m_1 + m_2} v_{2i} \qquad \ldots(3)$$

Similarly,

$$v_{2f} = \frac{m_2 - em_1}{m_1 + m_2} v_{2i} + \frac{(1+e)m_1}{m_1 + m_2} v_{1i} \qquad \ldots(4)$$

3.13. LINEAR MOMENTUM OF A SYSTEM OF PARTICLES

Consider a system of n particles of masses m_1, m_2, ..., m_n and velocities $\vec{v}_1$, $\vec{v}_2$, ..., $\vec{v}_n$ respectively. The particles may be interacting and have external forces acting on them. The linear momentum of the first particle is $m_1\vec{v}_1$, of the second particle is $m_2\vec{v}_2$ and so on.

For the system of n particles, the linear momentum of the system is defined to be the vector sum of momenta of all individual particles of the system.

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_n = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots + m_n\vec{v}_n$$

But $m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots + m_n\vec{v}_n = M\vec{V}$

$$\therefore \quad \vec{P} = M\vec{V} \qquad \ldots(1)$$

Thus, *the total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass.*

3.14. MOMENTUM CONSERVATION

Differentiating Eq. (1) with respect to time,

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{V}}{dt} = M\vec{A}$$

But $\quad M\vec{A} = \vec{F}_{ext.}$

where $\vec{F}_{ext.}$ represents the sum of all external forces acting on the particles of the system.

$$\therefore \quad \frac{d\vec{P}}{dt} = \vec{F}_{ext.} \qquad \ldots(2)$$

This is the statement of **Newton's second law extended to a system of particles.**

Suppose now, that the sum of external forces acting on a system of particles is zero. Then from Eq. (2)

$$\frac{d\vec{P}}{dt} = 0 \quad \text{or} \quad \vec{P} = \text{constant} \qquad \ldots(3)$$

Thus, when the total external force acting on a system of particles is zero, the total linear momentum of the system is constant. This is the **law of conservation of the total linear momentum** of a system of particles.

Rewriting Eq. (3),

$$M\vec{V} = \text{constant}$$

or $$\vec{V} = \text{constant} \qquad (\because \text{ M is constant.})$$

Thus, if the total external force acting on the system is zero, the centre of mass moves with a constant velocity *i.e.*, moves uniformly in a straight line like a free particle. This is Newton's first law of motion.

3.15. EXAMPLES OF MOTION OF CENTRE OF MASS

Following are examples of motion of centre of mass:

1. A projectile, following the usual parabolic trajectory, explodes into fragments midway in air. The forces leading to the explosion are internal forces. They contribute nothing to the motion of the centre of mass. The total external force, namely, the force of gravity acting on the body, is the same before and after the explosion. The centre of mass under the influence of the external force continues, therefore, along the same parabolic trajectory as it would have followed if there were no explosion.

In this illustration, the forces of explosion are all internal forces. These forces are exerted by part of the system on other parts of the system. These forces may change the momenta of all the individual fragments from the values they had when they made up the projectile. But the internal forces cannot change the total vector momentum of the system. It is

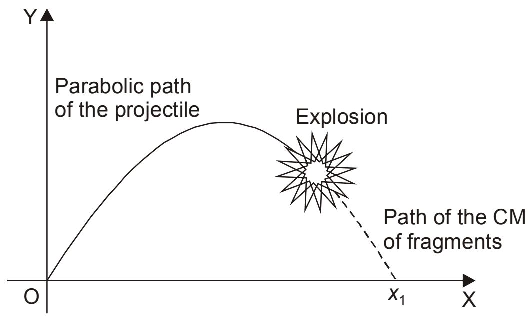


Fig. 3.8. The centre of mass of the fragments of the projectile continues along the same parabolic path which it would have followed if there were no explosion

only the external force which can change the total momentum of the system. In the given problem, the only external force is that due to gravity. The change in the total momentum of the system due to gravity is the same whether the shell explodes or not.

2. Consider the Earth-Moon system. Both the Earth and the Moon move in circles about their centre of mass, always being on opposite sides of it. The centre of mass moves along an elliptical path around the Sun. The forces of attraction between Earth and Moon are internal to the Earth-Moon system. On the other hand, the Sun's attraction of both Earth and Moon are external forces.

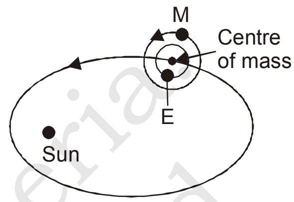


Fig. 3.9. Centre of mass of Earth-Moon system

3.16. RIGID BODIES AND ROTATIONAL MOTION

After having considered a system of particles which moves under the influence of internal and external forces, we can now take up the rotational motion of rigid body. **A rigid body is a body with a perfectly definite and unchanging shape.** *The geometrical shape and size of rigid body do not undergo any change during motion of rigid body.* A rigid body may be regarded as assembly of point masses. The mutual distances among different point masses do not change during the motion of the rigid body.

3.17. CENTRE OF MASS OF A RIGID BODY

The ***centre of mass*** *of a rigid body is a point whose position is fixed with respect to the body as a whole.* This point may or may not be within the body. The position of the centre of mass of a rigid body depends upon the following two factors.

(*i*) shape of the body **(*ii*)** distribution of mass in the body.

It is easy to locate the centre of mass of a symmetrical rigid body having uniform distribution of mass. In most of such cases, the centre of mass is at the geometrical centre.

Position of Centre of Mass of Some Regular Bodies

S. No.	Shape of body	Position of centre of mass
1.	Uniform rod	Centre of rod
2.	Plane rectangular or square lamina	Point of intersection of diagonals
3.	Plane triangular lamina	Point of intersection of the medians of triangle
4.	Uniform circular ring	Centre of ring
5.	Uniform circular disc	Centre of disc
6.	Uniform solid sphere	Centre of the solid sphere
7.	Uniform hollow sphere	Centre of the hollow sphere
8.	Uniform hollow cylinder	Midpoint of the axis of the hollow cylinder
9.	Uniform solid cylinder	Midpoint of the axis of the solid cylinder

3.18. LAW OF CONSERVATION OF LINEAR MOMENTUM

(*i*) Statement. ***If the vector sum of the external forces acting on a system is zero, then the total momentum of the system is conserved i.e., remains constant.***

The concept of conservation of momentum is particularly important in situations in which we have two or more interacting bodies. The law of conservation of momentum is a direct consequence of Newton's third law. This law does not depend on the detailed nature of the internal forces that act between the members of the system.

- *For any system of particles, the forces that the particles of the system exert on each other are called internal forces.*
- *The forces exerted on any part of the system by some object outside the system are called external forces.*
- *A system is said to be* ***isolated*** *if the net external force acting on the system is zero.*
- *A system is said to be* ***closed*** *if no particles enter or leave the system.*

For a closed, isolated system,

$$\vec{p} = \text{constant} \qquad \ldots(1)$$

$$\Rightarrow \qquad \vec{p}_i = \vec{p}_f \qquad \ldots(2)$$

The total linear momentum at some initial time t_i = total linear momentum at some later time t_f.

Equations (1) and (2) are vector equations. Each is equivalent to three equations corresponding to the conservation of linear momentum in three mutually perpendicular directions. Depending on the forces acting on a system, linear momentum might be conserved in one or two directions but not in all directions. *If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum of the system along that axis cannot change.*

(*ii*) Derivation of the law of conservation of momentum from Newton's second law of motion.

According to Newton's second law of motion, the time rate of change of momentum is equal to the applied force.

If the system is isolated, then $\vec{F} = 0$.

In that case, $\dfrac{d}{dt}(\vec{p}) = 0$

$\therefore \quad \vec{p}$ = constant

[Differential coefficient of an isolated constant is zero.]

This leads us to the following statement of the law of conservation of momentum.

"*In the absence of external forces, the total momentum of the system is conserved*".

(*iii*) Derivation of the law of conservation of momentum from Newton's third law of motion.

Consider an isolated system consisting of two bodies A and B of masses m_1 and m_2 respectively [Fig. 3.10]. Let the two bodies be moving along a straight line in the same direction. Let their respective velocities be $\vec{v}_{1i}$ and $\vec{v}_{2i}$ such that $\vec{v}_{1i}$ is greater than $\vec{v}_{2i}$. The two bodies will collide after some time. Let $\vec{v}_{1f}$ and $\vec{v}_{2f}$ be the velocities of A and B respectively after the collision.

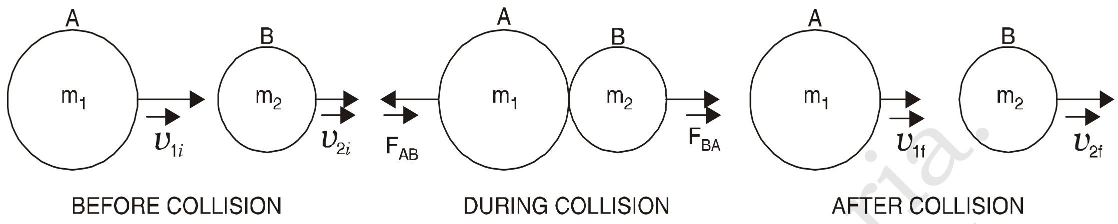


Fig. 3.10. One-dimensional collision

Before collision

Momentum of body A = $m_1\vec{v}_{1f}$; Momentum of body B = $m_2\vec{v}_{2i}$

$\therefore$ Total momentum of system = $m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$

After collision

Momentum of body A = $m_1\vec{v}_{1f}$; Momentum of body B = $m_2\vec{v}_{2f}$

$\therefore$ Total momentum of system = $m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Change in momentum of body A = $m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$

Change in momentum of body B = $m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$

During collision, the body A exerts an average force $\vec{F}_{BA}$ on body B. According to Newton's third law of motion, the body B will exert an average force $\vec{F}_{AB}$ on body A such that

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

Let t be the duration of collision.

Then, impulse acting on B = $\vec{F}_{BA}\,t$; Impulse acting on A = $\vec{F}_{AB}\,t$

But impulse = change in momentum

$\therefore$ $\vec{F}_{BA}\,t = m_2\vec{v}_{2i} - m_2\vec{v}_{2f}$ and $\vec{F}_{AB}\,t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$

But $\vec{F}_{BA}\,t = -\vec{F}_{AB}\,t$

$\therefore$ $m_2\vec{v}_{2f} - m_2\vec{v}_{2i} = -(m_1\vec{v}_{1f} - m_1\vec{v}_{1i})$

or $m_2\vec{v}_{2f} + m_1\vec{v}_{1f} = m_1\vec{v}_{1i} + m_2\vec{v}_{2i}$

So, total momentum of system after collision is equal to the total momentum of system before collision.

This leads to the following statement of the law of conservation of momentum.

"The total vector sum of the momenta of bodies, in an isolated system, along any straight line remains conserved and is unchanged due to the mutual action and reaction between the bodies in the system."

This law is universal. It is true not only for collisions between astronomical bodies but also for collisions between atomic particles.

3.19. APPLICATIONS/ILLUSTRATIONS OF LAW OF CONSERVATION OF MOMENTUM

(*i*) Recoil of a Gun. Let the gun and the bullet in its barrel constitute one isolated system.

To begin with, both the gun and the bullet are at rest. So, the momentum of the system, before firing, is zero.

When the bullet is fired, it moves in the forward direction and the gun kicks backward.

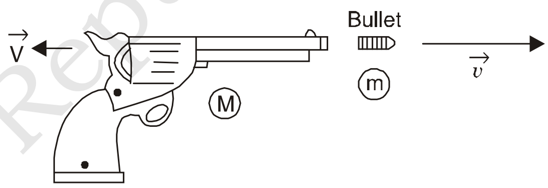


Fig. 3.11. Recoil of gun

Let, m = mass of bullet ; M = mass of gun ; $\vec{v}$ = velocity of bullet ; $\vec{V}$ = velocity of gun.

Total momentum of system after firing = $M\vec{V} + m\vec{v}$

No external forces have acted on the system. So, law of conservation of momentum can be applied.

$$\therefore \qquad M\vec{V} + m\vec{v} = 0 \quad \text{or} \quad M\vec{V} = -m\vec{v}$$

or
$$\vec{V} = -\frac{m}{M}\vec{v}$$

The negative sign shows that the velocity $\vec{V}$ of recoil is opposite to the velocity of the bullet, *i.e.,* if the bullet moves in the forward direction, the gun moves in the backward direction.

The mass M of the gun is very large as compared to the mass m of the bullet. So, the velocity of recoil is very small as compared to the velocity of the bullet.

***(ii)* Machine Gun firing Bullets.** Suppose a machine gun mounted on a car is firing n bullets in time t. Let m and $\vec{v}$ be the mass and velocity respectively of each bullet [Fig. 3.12].

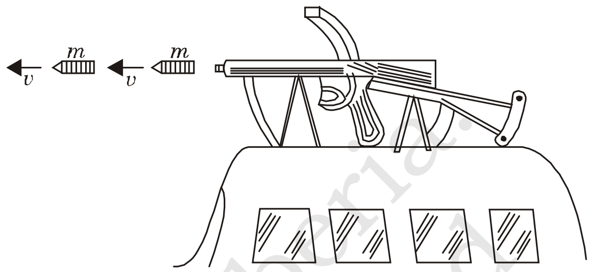


Fig. 3.12. Machine gun firing bullets

Total momentum in the forward direction = $n \times m\vec{v}$

The reaction of this momentum will be in the backward direction. This reaction will set the car in motion to the right. In order to hold the car in position, the accelerator of the car shall have to be suitably pressed. The applied force $\vec{F}$ should be such that

$$\vec{F}\,t = -\,nm\vec{v} \quad \text{[Impulse = change of momentum]}$$

***(iii)* Explosion of a Bomb.** Suppose a bomb is at rest as shown in Fig 3.13 (*a*). Its momentum will be zero. Let the bomb explode into five fragments of masses m_1, m_2, m_3, m_4 and m_5 [Fig. 3.13 (*b*)].

Let their respective velocities be $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ and $\vec{v}_5$. Then their respective momenta will be given by

$$\vec{p}_1 = m_1\vec{v}_1,\ \vec{p}_2 = m_2\vec{v}_2,\ \vec{p}_3 = m_3\vec{v}_3,\ \vec{p}_4 = m_4\vec{v}_4 \text{ and } \vec{p}_5 = m_5\vec{v}_5$$

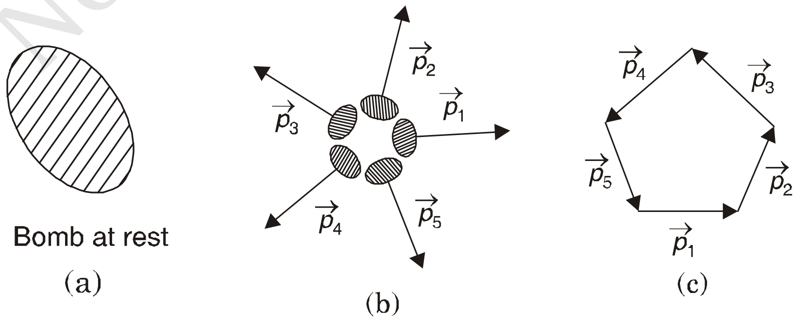


Fig. 3.13. Explosion of a bomb

No external force has acted on the system. Therefore, the law of conservation of momentum can be applied.

$\therefore$ Momentum after explosion = Momentum before explosion

$$\therefore \qquad \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4 + \vec{p}_5 = \vec{0}$$

The sum of the five momenta vectors is zero. So, they can be represented both in magnitude and direction by the five sides of a closed polygon, all taken in the same order. This is shown in Fig. 3.13(*c*).

If the bomb explodes into two fragments of equal masses, then the fragments will move with equal speeds in opposite directions.

3.20. LAW OF CONSERVATION OF MOMENTUM AND CENTRE OF MASS

Consider an isolated system consisting of n particles of masses m_1, m_2, m_3, ..., m_n. Let $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, ..., $\vec{v}_n$ be their respective velocities.

The total linear momentum $\vec{P}$ of the system is equal to the vector sum of the linear momenta of all the particles in the system.

$$\text{Then } \vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots\ldots + m_n\vec{v}_n$$

$$\text{or} \quad \vec{P} = (m_1 + m_2 + m_3 + \ldots\ldots + m_n)\left\{\frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots.. + m_n\vec{v}_n}{m_1 + m_2 + m_3 + \ldots.. + m_n}\right\}$$

But $m_1 + m_2 + m_3 + \ldots\ldots + m_n = \text{M}$ (total mass of system)

$$\text{and} \quad \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots.. + m_n\vec{v}_n}{m_1 + m_2 + m_3 + \ldots.. + m_n} = \vec{V}_{c.m.}$$

where $\vec{V}_{c.m.}$ is the velocity of centre of mass of the system.

$$\therefore \qquad \vec{P} = \text{M}\vec{V}_{c.m.}$$

Since the given system is isolated therefore no external force will act. According to the law of conservation of momentum, the total momentum $\vec{P}$ should be constant.

$$\therefore \qquad M\vec{V}_{c.m.} = \text{constant.}$$

CONCLUSION

When no external force acts on the system, the centre of mass of the system has a constant velocity.

Example 5. *A gun weighing 10 kg fires a bullet of 30 g with a velocity of 330 m s^{-1}. With what velocity does the gun recoil ? What is the combined momentum of the gun and bullet before firing and after firing?*

Solution. Mass of gun, M = 10 kg

Mass of bullet, m = 30 g = 0.03 kg

Velocity of bullet, v = 330 m s^{-1}

Velocity of recoil, V = ?

In magnitude, momentum of gun = momentum of bullet

$$\therefore \qquad MV = mv \quad \text{or} \quad V = \frac{mv}{M}$$

$$\therefore \qquad V = \frac{0.03 \times 330}{10} \text{ m s}^{-1} = \mathbf{0.99 \text{ m s}^{-1}}$$

Combined momentum of gun and bullet before firing is zero. Since no external force has acted therefore momentum must be conserved. So, the combined momentum of gun and bullet after firing is also **zero.**

Example 6. *A hunter has a machine gun that can fire 50 g bullets with a velocity of 900 m s^{-1}. A 40 kg tiger springs at him with a velocity of 10 m s^{-1}. How many bullets must the hunter fire into the tiger in order to stop him in his track?*

Solution.

Mass of bullet, m = 50 g = 0.05 kg

Velocity of bullet, v = 900 m s^{-1}

Mass of tiger, M = 40 kg

Velocity of tiger, V = 10 m s^{-1}

Let n be the number of bullets required to be pumped into the tiger to stop him in his track.

If the bullets and the tiger are supposed to constitute one isolated system, then the magnitude of the momentum of n bullets should be equal to the magnitude of momentum of the tiger.

$$\therefore \qquad n \times m\,v = \text{MV} \quad \text{or} \quad n = \frac{\text{MV}}{mv}$$

$$\therefore \qquad n = \frac{40 \times 10}{0.05 \times 900} = 8.89 \approx \mathbf{9}$$

3.21. MOMENT OF INERTIA

(*i*) ***Moment of inertia*** *of a rigid body about a fixed axis is defined as the sum of the products of the masses of all the particles constituting the body and the squares of their respective distances from the axis of rotation.* It is a scalar quantity.

Let YY′ be the axis about which the rigid body is rotating [Fig. 3.14]. Let the body be composed of n particles of masses m_1, m_2, ..., m_n. Let r_1, r_2, ..., r_n be their respective distances from the axis of rotation. The moment of inertia of the rigid body about the given axis YY′ is given by

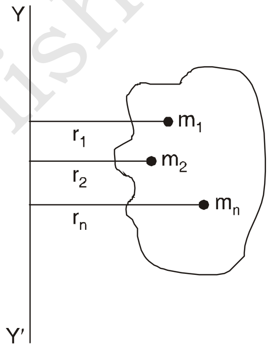


Fig. 3.14. Moment of inertia of a rigid body

$$\text{I} = m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2 = \sum_{i=1}^{n} m_1 r_1^2$$

(*ii*) In cgs system, the unit of moment of inertia is g cm^2. In SI, moment of inertia is measured in kg m^2.

(*iii*) Moment of inertia depends on the following **factors:**

1. Mass of the body.
2. Position of the axis of rotation.
3. Distribution of mass about the axis of rotation.

3.22. ANGULAR MOMENTUM OF A PARTICLE

(*a*) The rotational analogue of momentum is moment of momentum. It is also referred to as angular momentum. This quantity is a measure of the twisting or turning effect associated with the momentum of the particle.

The ***angular momentum (or moment of momentum)*** *about an axis of rotation is a vector quantity, whose magnitude is equal to the product of the magnitude of momentum and the perpendicular distance of the line of action of momentum from the axis of rotation and its direction is perpendicular to the plane containing the momentum and the perpendicular distance.*

Fig. 3.15 shows a particle having linear momentum $\vec{p}$. Its position vector with reference to point O is $\vec{r}$. The perpendicular distance of the line of action of momentum from O is d. The angular momentum of the particle about an axis passing through O and perpendicular to the plane of the paper is given by:

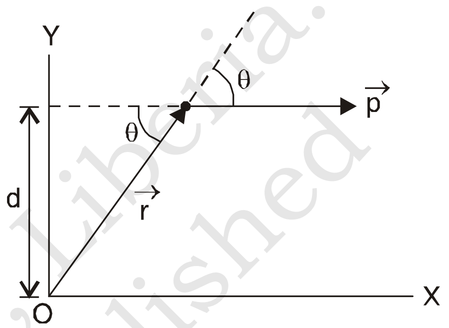


Fig. 3.15

$$L = pd$$

The cgs and SI units of L are g cm^2 s^{-1} and kg m^2 s^{-1} respectively. Its dimensional formula is [ML^{2}T^{-1}].

(*b*) Angular Momentum in Vector Notation. Fig. 3.16 shows position vector $\vec{r}$ and momentum $\vec{p}$ of a particle P in XOY plane. The angular momentum of the particle P with respect to the origin O is given by:

$$\vec{L} = \vec{r} \times \vec{p}$$

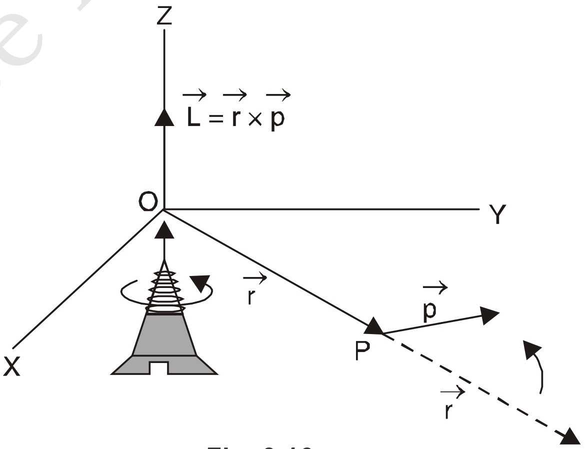


Fig. 3.16

The direction of $\vec{L}$ is obtained by applying the right-hand rule for the vector product of two vectors. In this case, $\vec{L}$ acts along OZ.

The angular momentum is taken as positive for anti-clockwise rotation and negative for clockwise rotation.

The magnitude of $\vec{L}$ is given by, $\quad L = rp \sin \theta \qquad \ldots(1)$

where r is the magnitude of the position vector $\vec{r}$ *i.e.*, the length OP, p is the magnitude of momentum $\vec{p}$ and θ is the angle between $\vec{r}$ and $\vec{p}$ as shown.

Now, $\quad \sin \theta = \dfrac{d}{r} \quad$ or $\quad d = r \sin \theta$

From eqn. (1), $\quad L = p\,(r \sin \theta) = pr_{\perp} = pd$

Again, $\quad L = r\,(p \sin \theta) = rp_{\perp} = rp_{\theta}$

Special Cases

(*i*) If $r = 0$, then L = 0. A particle at O has zero angular momentum about O.

(*ii*) If $\theta = 0°$ or $180°$, then $\sin \theta = 0$.

$\therefore \quad L = rp \sin \theta = 0$

In this case, the line of action of the momentum passes through the point O. Thus, if the line of action of momentum passes through point O, the angular momentum is zero.

(*iii*) If $\theta = 90°$, then $\sin \theta = \sin 90° = 1$ (max. value). So, L is maximum.

$$L_{max.} = rp$$

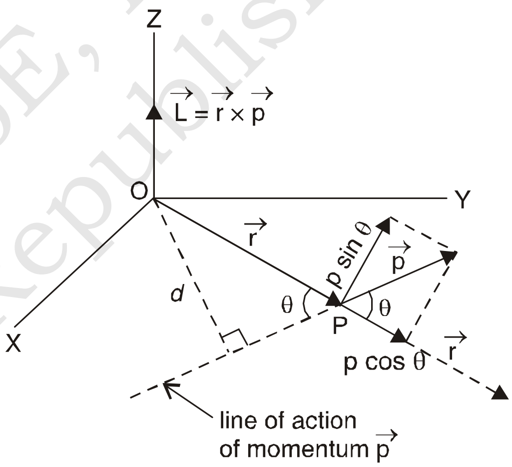


Fig. 3.17

3.23. RELATION BETWEEN ANGULAR MOMENTUM AND TORQUE

We know that, $\vec{L} = \vec{r} \times \vec{p}$

Differentiating both sides w.r.t. t, we get

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

or
$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \vec{v} \times \vec{p}$$

or $$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} \quad [\because \vec{v} \times \vec{p} = \vec{v} \times m\vec{v} = m(\vec{v} \times \vec{v}) = 0]$$

According to Newton's second law of motion, $\frac{d\vec{p}}{dt} = \vec{F}$

$$\therefore \quad \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} \quad \text{or} \quad \frac{d\vec{L}}{dt} = \vec{\tau} \qquad \ldots(1)$$

So, *the time rate of change of the angular momentum of a particle is equal to the torque acting on it.* This result is the rotational analogue of the statement—"The time rate of change of the linear momentum of a particle is equal to the force acting on it."

Like all vector equations, equation (1) is equivalent to three scalar equations, namely, $\tau_x = \frac{dL_x}{dt}$, $\tau_y = \frac{dL_y}{dt}$ and $\tau_z = \frac{dL_z}{dt}$

So, the x-component of the applied torque is given by x-component of the change with time of the angular momentum. Similar results hold for the y and z-directions.

3.24. TORQUE AND ANGULAR MOMENTUM FOR A SYSTEM OF PARTICLES

The total angular momentum of a system of particles about a given point is the vector sum of the angular momenta of individual particles about the given point. For a system of n particles,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \ldots\ldots + \vec{L}_n = \sum_{i=1}^{n} \vec{L}_i$$

The angular momentum of the i^{th} particle is given by

$$\vec{L}_i = \vec{r}_i \times \vec{p}_i$$

where $\vec{r}_i$ is the position vector of the i^{th} particle with respect to the given origin and $\vec{p}_i$ $(= m_i\vec{v}_i)$ is the linear momentum of the i^{th} particle.

Now, $$\vec{L} = \sum_{i=1}^{n} \vec{L}_i = \sum_{i=1}^{n} \vec{r}_i \times \vec{p}_i$$

This is a generalisation of the definition of angular momentum for a single particle to a system of particles.

Now, $$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\Sigma \vec{L}_i) = \sum_{i=1}^{n} \frac{d\vec{L}_i}{dt} = \sum_{i=1}^{n} \vec{\tau}_i$$

where $\vec{\tau}_i$ is the torque acting on the i^{th} particle;

3.25. EQUILIBRIUM OF RIGID BODIES

A rigid body is said to be in mechanical equilibrium if both its linear momentum and angular momentum are not changing with time, or equivalently the body has neither linear acceleration nor angular acceleration.

A rigid body such as a chair, a bridge or building is said to be in equilibrium if both the linear momentum and the angular momentum of the rigid body have a constant value. When a rigid body is in equilibrium, the linear acceleration of its centre of mass is zero. Also, the angular acceleration of the rigid body about any fixed axis in the reference frame is zero.

For the equilibrium of a rigid body, it is not necessary that the rigid body is at rest. However, *if the rigid body is at rest, then the equilibrium of the rigid body is called* ***static equilibrium.***

(*i*) First Condition for Equilibrium. The translational motion of the centre of mass of a rigid body is governed by the following equation :

$$\Sigma \vec{F}_{ext.} = \frac{d}{dt}(\vec{p})$$

A rigid body is said to be in translational equilibrium if it remains at rest or moves with a constant velocity in a particular direction.

Here $\Sigma \vec{F}_{ext.}$ is the vector sum of all the external forces that act on the rigid body.

For equilibrium, $\vec{p}$ must have a constant value. $\therefore \frac{d}{dt}(\vec{p}) = 0$

$$\therefore \qquad \Sigma \vec{F}_{ext.} = 0$$

This vector equation is equivalent to three scalar equations:

$$\sum_{i=1}^{n} F_{ix} = 0, \sum_{i=1}^{n} F_{iy} = 0, \sum_{i=1}^{n} F_{iz} = 0 \quad \ldots(1)$$

This leads us to ***the first condition for the equilibrium of rigid bodies.***

"The vector sum of all the external forces acting on the rigid body must be zero".

(*ii*) Second Condition for Equilibrium. The rotational motion of a rigid body is governed by the following equation:

$$\Sigma \vec{\tau}_{ext.} = \frac{d\vec{L}}{dt}$$

Here $\Sigma \vec{\tau}_{ext.}$ represents the vector sum of all the external torques that act on the body.

For equilibrium, $\vec{L}$ must have a constant value. $\therefore \frac{d}{dt}(\vec{L}) = 0$

$$\therefore \quad \Sigma \vec{\tau}_{ext.} = 0$$

This vector equation can be written as three scalar equations:

$$\sum_{i=1}^{n} \tau_{ix} = 0, \sum_{i=1}^{n} \tau_{iy} = 0, \sum_{i=1}^{n} \tau_{iz} = 0 \quad \ldots(2)$$

This leads us to ***the second condition for the equilibrium of rigid bodies.***

"The vector sum of all the external torques acting on the rigid body must be zero."

3.26. SOME APPLICATIONS AND EXAMPLES OF THE LAW OF CONSERVATION OF ANGULAR MOMENTUM

1. The angular velocity of a planet around the Sun increases when it comes near the Sun.

When a planet revolving around the Sun in an elliptical orbit comes near the Sun, the moment of inertia of the planet about the Sun decreases. In order to conserve angular momentum, the angular velocity shall increase. Similarly, when the planet is away from the Sun, there will be a decrease in the angular velocity.

2. The speed of the inner layers of the whirlwind in a tornado is alarmingly high.

In a tornado, the moment of inertia of air will go on decreasing as the air moves towards the centre. This will be accompanied by an increase in angular velocity such that the angular momentum is conserved.

3. A diver jumping from a spring board performs somersaults in air.

When a diver jumps from spring board, he curls his body by rolling in his arms and legs. This decreases moment of inertia and hence increases angular velocity. He then performs somersaults. As the diver is about to touch the surface of water, he stretches out his limbs. By so doing, he increases his moment of inertia, thereby reducing his angular velocity.

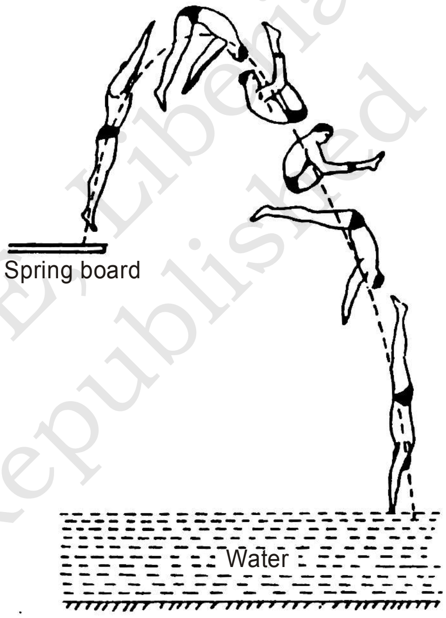


Fig. 3.18. Diver performing somersaults

4. A ballet dancer can vary her angular speed by outstretching her arms and legs.

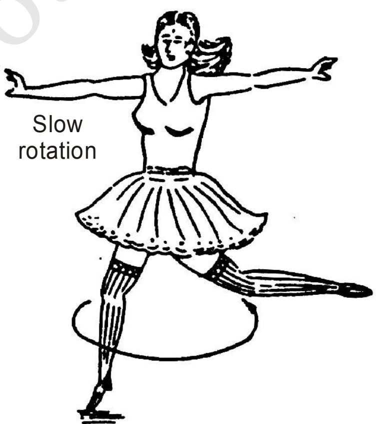


Fig. 3.19. Ballet dancer making use of law of conservation of angular momentum

A ballet dancer [Fig. 3.19] makes use of the law of conservation of angular momentum to vary her angular speed. Suppose a ballet dancer is rotating with her legs and arms stretched outwards. When she suddenly folds her arms and brings the stretched leg close to the other leg, her angular velocity increases on account of decrease in moment of inertia [Fig. 3.19].

5. A man carrying heavy weights in his hands and standing on a rotating table can vary the speed of the table.

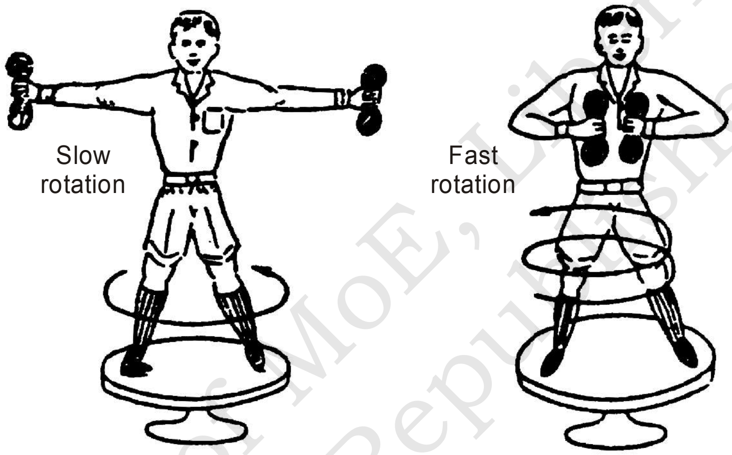


Fig. 3.20

Suppose a man is standing on a rotating table with his arms outstretched. Suppose he is holding heavy weights in his hands. When the man suddenly folds his arms, his angular velocity increases on account of the decrease in moment of inertia [Fig. 3.20].

3.27. LAWS OF ROTATIONAL MOTION

I. Unless an external torque is applied to it, a body in a state of rest or uniform rotational motion about its fixed axis of rotation remains unchanged.

II. The rate of change of angular momentum of a body about a fixed axis of rotation is directly proportional to the torque applied and takes place in the direction of the torque.

III. When a torque is applied by one body on another, an equal and opposite torque is applied by the latter on the former about the same axis of rotation.

REVIEW EXERCISES

Do the review exercises in your notebook.

A. Multiple Choice Questions

1. A ball of mass M falls from a height h on a floor for which the coefficient of restitution is e. The height attained by the ball after two rebounds is

(*a*) $e^2 h$ (*b*) eh^2

(*c*) $e^4 h$ (*d*) h/e^4.

2. Consider the following two statements:

A. Linear momentum of a system of particle is zero. Then

B. Kinetic energy of a system of particles is zero. Then

(*a*) A does not imply B but B implies A.

(*b*) A implies B and B implies A.

(*c*) A does not imply B and B does not imply A.

(*d*) A implies B but B does not imply A.

3. A spring of spring constant 5×10^3 N m^{-1} is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is

(*a*) 25.00 N m (*b*) 6.25 N m

(*c*) 12.50 N m (*d*) 18.75 N m.

4. A neutron makes a head-on elastic collision with a stationary deuteron. The fractional energy loss of the neutron in the collision is

(*a*) 16/81 (*b*) 8/9

(*c*) 8/27 (*d*) 2/3.

5. A stationary particle explodes into two particles of masses m_1 and m_2 which move in opposite directions with velocities v_1 and v_2. The ratio of their kinetic energies E_1/E_2 is

(*a*) m_2/m_1 (*b*) m_1/m_2

(*c*) 1 (*d*) m_1v_2/m_2v_1.

6. A body of mass m has a kinetic energy equal to one-fourth kinetic energy of another body of mass $m/4$. If the speed of the heavier body is increased by 4 m s^{-1}, its new kinetic energy equals the original kinetic energy of the lighter body. The original speed of the heavier body in m s^{-1} is

(*a*) 8 (*b*) 6

(*c*) 4 (*d*) 2.

7. A toy gun has a spring of force constant k. After charging the spring by compressing it through a distance of x, the toy releases a shot of mass m vertically upwards. Then the shot will travel a vertical height of

(*a*) $\dfrac{2mg}{kx^2}$ (*b*) $\dfrac{kx^2}{mg}$

(*c*) $\dfrac{kx}{mg}$ (*d*) $\dfrac{kx^2}{2mg}$.

8. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

(*a*) x^2 (*b*) e^x

(*c*) x (*d*) $\log_e x$.

9. An automobile travelling with a speed of 60 km h^{-1}, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i.e.*, at 120 km h^{-1}, the stopping distance will be

(*a*) 20 m (*b*) 40 m

(*c*) 60 m (*d*) 80 m.

10. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table ?

(*a*) 7.2 J (*b*) 3.6 J

(*c*) 120 J (*d*) 1200 J.

B. Fill in the Blanks

1. A body of mass 3 kg is under a constant force which causes a displacement s (in m) in it, given by the relation $s = \dfrac{1}{3}t^2$, where t is in second. Work done by the force in 2 s is __________ .

2. A 2 kg block slides on a horizontal floor with a speed of 4 m s^{-1}. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N m^{-1}. The spring compresses by __________ .

3. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is __________ .

4. A bread gives a boy of mass 40 kg an energy of 21 kJ. If the efficiency is 28%, then the height which can be climbed by him using this energy is nearly __________ .

5. A windmill converts wind energy into electrical energy. If v is the wind speed, electrical power output is proportional to __________ .

C. Very Short Answer Questions

1. A rough inclined plane is placed on a cart moving with a constant velocity u on horizontal ground. A block of mass M rests on the incline. Is any work done by force of friction between the block and incline? Is there then a dissipation of energy?

2. Why is electrical power required at all when the elevator is descending? Why should there be a limit on the number of passengers in this case?

3. A body is being raised to a height h from the surface of earth. What is the sign of work done by
   (*a*) applied force
   (*b*) gravitational force?

4. Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2 m.

5. A body falls towards Earth in air. Will its total mechanical energy be conserved during the fall? Justify.

D. Short Answer Questions

1. The potential energy of two atoms separated by a distance x is given by U = $-\dfrac{A}{x^6}$ where A is a positive constant. Find the force exerted by one atom on another atom.

2. A ball, dropped from a height of 8 m, hits the ground and bounces back to a height of 6 m only. Calculate the fractional loss in kinetic energy.

3. A particle of moving in a circle with centripetal force $-\dfrac{K}{r^2}$. What is the total energy associated?

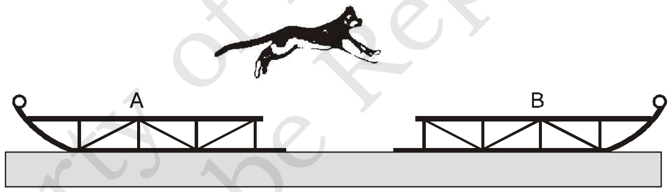
4. A particle of mass m strikes on ground with angle of incidence 45°. If coefficient of restitution $e = 1/\sqrt{2}$, find the velocity of reflection and angle of reflection?

5. A body of mass m falls from a height h and collides with another body of same mass. After collision, the two bodies combine and move through distance d till they come to rest. Find the work done against the resistive force.

E. Long Answer Questions

1. A rubber ball of mass 50 g falls from a height of 1 m and rebounds to a height of 50 cm. Calculate the impulse and the average force between the ball and the ground, if the time during which they are in contact was 0.1 second.

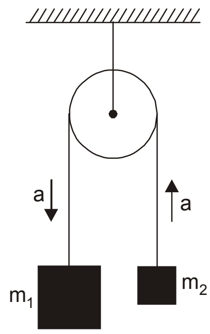
2. Two 22.7 kg ice sleds A and B are placed a short distance apart, one directly behind the other, as shown in figure below. A 3.63 kg cat, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of 3.05 m s^{-1} relative to the ice. Find the final speeds of the two sleds.



3. A bullet of mass 7 g is fired into a block of metal weighing 7 kg. The block is free to move. After the impact, the velocity of the bullet and the block is 0.7 m s^{-1}. What is the initial velocity of the bullet ?

4. A block of mass m moving at speed v collides with another block of mass 2 m at rest. The lighter block comes to rest after the collision. Find the coefficient of restitution.

5. Two bodies of masses m_1 and m_2 ($< m_1$) are connected to the ends of a massless cord and allowed to move as shown. The pulley is both massless and frictionless. Determine the acceleration of the centre of mass.



SEMESTER-2 (Period-IV)

TOPIC

4

Heat

P11CH4

4.1. INTRODUCTION

We all have common-sense notions of heat and temperature. Temperature is a measure of 'hotness' of a body. A kettle with boiling water is hotter than a box containing ice. In physics, we need to define the notion of heat, temperature, etc., more carefully. In this chapter, you will learn what heat is and how it is measured. You will also learn what happens when water boils or freezes, and its temperature does not change during these processes even though a great deal of heat is flowing into or out of it.

4.2. TEMPERATURE AND HEAT

Temperature is a relative measure, or indication, of hotness or coldness. A hot utensil is said to have a high temperature, and ice cube to have a low temperature. An object that has a higher temperature than another object is said to be hotter. Note that hot and cold are relative terms, like tall and short. We can perceive temperature by touch. However, this temperature sense is somewhat unreliable and its range is too limited to be useful for scientific purposes.

We know from experience that a glass of ice-cold water left on a table on a hot summer day eventually warms up whereas a cup of hot tea on the same table cools down. It means that when the temperature of body, ice-cold water or hot tea in this case, and its surrounding medium are different, heat is exchanged between the system and the surrounding medium, until the body and the surrounding medium are at the same temperature. We also know that in the case of cold water, heat flows from the environment to the glass tumbler whereas in the case of hot tea, it flows from the cup of hot tea to the environment. ***Heat*** *is the form of energy transferred between two (or more) systems or a system and its surroundings by virtue of temperature difference.*

The SI unit of heat energy transferred is expressed in joule (J). SI unit of temperature is kelvin (K). The commonly used unit of temperature is °C.

The cgs or practical unit of heat is a calorie. *One calorie is the amount of heat required to raise the temperature of 1 gram of water from 14.5°C to 15.5°C.*

1 calorie = 4.186 joule ; 4.2 joule

4.3. MEASUREMENT OF TEMPERATURE

Thermometer is an instrument which is used to measure the temperature of a body.

Many physical properties of materials change sufficiently with temperature to be used as the basis for constructing thermometers. The commonly used property is variation of the volume of a liquid with temperature. Mercury and alcohol are the liquids used in most liquid-in-glass thermometers.

Thermometers are calibrated so that a numerical value may be assigned to a given temperature. For the definition of any standard scale, two fixed reference points are needed. The ice point and the steam point of water are two convenient fixed points and are known as the freezing and boiling points. These two points are the temperature at which pure water freezes and boils under standard pressure.

The two familiar temperature scales are the Fahrenheit temperature scale and the Celsius temperature scale. The ice and steam point have values 32°F and 212°F respectively, on the Fahrenheit scale and that of 0°C and 100°C on the Celsius scale. On the Fahrenheit scale, there are 180 equal intervals between two reference points, and on the Celsius scale, there are 100.

A relationship for converting between the two scales may be obtained from a graph of Fahrenheit temperature (t_F) versus Celsius temperature (t_C) in a straight line (Fig. 4.1), whose equation is

$$\frac{t_F - 32}{180} = \frac{t_C}{100}$$

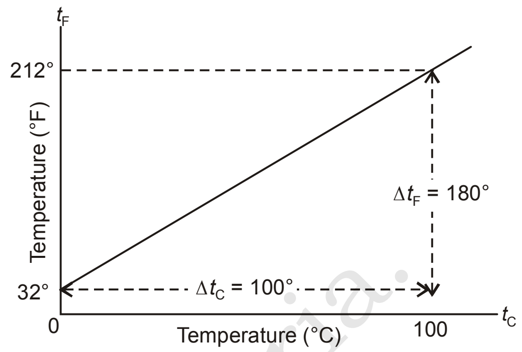


Fig. 4.1. A plot of Fahrenheit temperature (t_F) versus Celsius temperature (t_C)

Example 1. *What is the temperature at which we get the same reading on both the centigrade and Fahrenheit scales?*

Solution. If t is the required temperature, then

$$\frac{t}{100} = \frac{t-32}{180}$$

On simplification, $t = -40$

So, the required temperature is **– 40°C** or **– 40°F.**

4.4. IDEAL GAS EQUATION AND ABSOLUTE SCALE OF TEMPERATURE

Liquid-in-glass thermometers show different readings for temperatures other than the fixed points because of differing expansion properties. A thermometer that uses a gas, however, gives the same readings regardless of which gas is used. The variables that describe the behaviour of a given quantity (mass) of gas are pressure, volume, and temperature (P, V and T) (where T = t + 273.15; t is the temperature in °C). When temperature is held constant, the pressure and volume of a quantity of gas are related as PV = constant. This relationship is known as Boyle's law, after Robert Boyle (1627–1691) the English Chemist who discovered it. When the pressure is held constant, the volume of a quantity of the gas is related to the temperature as V/T = constant. This relationship is known as Charles' law, after the French scientist Jacques Charles (1747–1823). Low density gases obey these laws, which may be combined into a single relationship.

Notice that since PV = constant and V/T = constant for a given quantity of gas, then PV/T should also be a constant. This relationship is known as ideal gas law. It can be written in a more general form that applies not just to a given quantity of a single gas but to any quantity of any dilute gas and is known as **ideal-gas equation :**

$$\frac{PV}{T} = \mu R$$

or $$PV = \mu RT \quad \ldots(1)$$

where μ is the number of moles in the sample of gas and R is called universal gas constant :

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

In Eq. (1), we have learnt that the pressure and volume are directly proportional to temperature : $PV \propto T$. This relationship allows a gas to be used to measure temperature in a constant volume gas thermometer. Holding the volume of a gas constant, it gives $P \propto T$. Thus, with a constant volume gas thermometer, temperature is read in terms of pressure. A plot of pressure versus temperature gives a straight line in this case, as shown in Fig. 4.2.

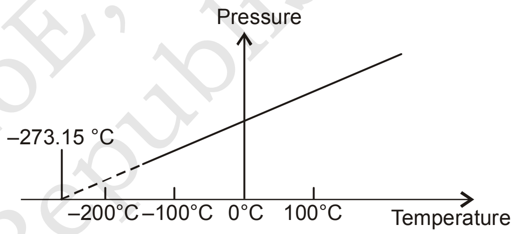


Fig. 4.2 Pressure versus temperature of a low density gas kept at constant volume

However, measurements on real gases deviate from the values predicted by the ideal gas law at low temperature. But the relationship is linear over a large temperature range, and it looks as though the pressure might reach zero with decreasing temperature if the gas continued to be a gas. The absolute minimum temperature for an ideal gas, therefore, inferred by extrapolating the straight line to the axis, as in Fig. 4.3. This temperature

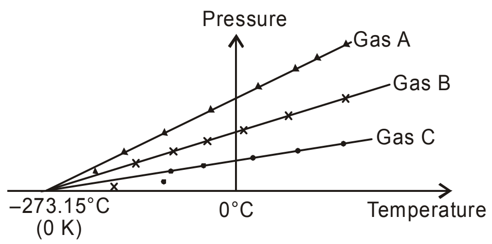


Fig. 4.3. A plot of pressure versus temperature and extrapolation of lines for low density gases indicates the same absolute zero temperature

is found to be – 273.15°C and is designated as **absolute zero.** Absolute zero is the foundation of the Kelvin temperature scale or absolute scale temperature named after the British scientist Lord Kelvin (1824–1907). The unit of temperature on this scale is written as K. On this scale, – 273.15°C is taken as the zero point, that is 0 K (Fig. 4.4).

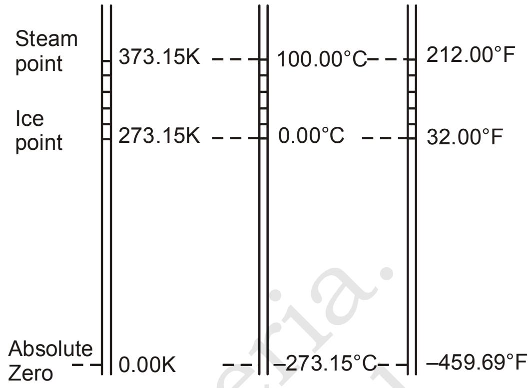


Fig. 4.4. Comparision of the Kelvin, Celsius and Fahrenheit temperature scales

The size of the unit for Kelvin temperature is the same Celsius degree, so temperature on these scales are related by

$$T = t_C + 273.15$$

4.5. SPECIFIC HEAT CAPACITY

(*i*) Heat capacity. The quantity of heat required to warm a given substance depends on its mass, *m*, the change in temperature, ΔT and the nature of substance. The change in temperature of a substance, when a given quantity of heat is absorbed or rejected by it, is characterised by a quantity called the **heat capacity** of that substance. We define heat capacity, S of a substance as

$$S = \frac{\Delta Q}{\Delta T}$$

where ΔQ is the amount of heat supplied to the substance to change its temperature from T to T + ΔT.

(*ii*) Specific heat capacity. If equal amount of heat is added to equal masses of different substances, the resulting temperature changes will not be the same. It implies that every substance has a unique value for the amount of heat absorbed or rejected to change the temperature of unit mass of it by one unit. This quantity is referred to as the **specific heat capacity** of the substance.

If ΔQ stands for the amount of heat absorbed or rejected by a substance of mass *m* when it undergoes a temperature change ΔT, then the specific heat capacity, of that substance is given by

$$s = \frac{S}{m} = \frac{1}{m}\frac{\Delta Q}{\Delta T} \qquad ...(1)$$

The **specific heat capacity** is the property of the substance which determines the change in the temperature of the substance (undergoing no phase change) when a given quantity of heat is absorbed (or rejected) by it. **It is defined as the amount of heat per unit mass absorbed or rejected by the substance to change its temperature by one unit.** It depends on the nature of the substance and its temperature.

The specific heat of a material is not constant. It depends on the location of the temperature interval. Equation (1) gives only the average value of specific heat capacity in the temperature range of ΔT.

The SI unit of specific heat capacity is J kg^{-1} K^{-1}.

From equation (1), $\Delta Q = ms\,\Delta T$

In differential notation,

$$dQ = ms\,dT$$

The heat required to increase the temperature of a body of mass m from T_i to T_f is given by

$$Q = m\int_{T_i}^{T_f} s\,dT$$

Equation (1) does not define specific heat capacity uniquely. We must specify the conditions under which heat is supplied to the system.

(*iii*) Molar specific heat capacity. It is often convenient to use the mole to describe the amount of substance. By definition, one mole of any substance is a quantity of matter such that its mass in gram is numerically equal to the molecular mass M (often called molecular weight). To calculate the number μ of moles, we divide the mass m in gram by the molecular mass M.

So, $$\mu = \frac{m}{M} \quad \text{or} \quad m = \mu M$$

If the amount of substance is specified in terms of moles μ, instead of mass m in kg, we can define heat capacity per mole of the substance by,

$$C = \frac{S}{\mu} = \frac{1}{\mu}\frac{\Delta Q}{\Delta T}$$

where C is known as **molar specific heat capacity** of the substance. Like S, C also depends on the nature of the substance and its temperature. The SI unit of molar specific heat capacity is J mol^{-1} K^{-1}.

4.6. DULONG AND PETIT LAW (SPECIFIC HEAT OF SOLIDS)

In 1819, two French physicists Dulong and Petit observed that the average molar specific heat of all metals, except the very lightest, is approximately constant and equal to nearly 3R = 6 cal mol^{-1} K^{-1} = 25 J mol^{-1} °C^{-1}.

Although the law is only an approximate one, it conveys a very important idea. **Nearly the same amount of heat is required per molecule to raise the temperature of metals by a given amount.** Thus, the heat required to raise the temperature of a sample of metal depends only on how many molecules the sample contains, and not on the mass of an individual molecule. One mole of each metal contains same number of atoms (= Avogadro's number). So, the molar specific heat of all metals at room temperature is nearly constant. This is a property of matter which is directly related to its molecular structure.

4.7. VARIATION OF SPECIFIC HEAT OF SOLIDS WITH TEMPERATURE

According to Dulong and Petit's law, the molar specific heat of every solid *must* come out to be equal to 6 cal. But the result is very nearly 6 cal and not exactly 6 cal. Also when we perform the experiment at various temperatures, we note that the specific heat is not constant quantity. Instead, it varies with temperature as shown in Fig. 4.5. At T = 0 K, C_v tends to be zero. With rise in temperature, C_v increases. At a specific temperature depending upon the nature of the material, C_v becomes constant (= 3R).

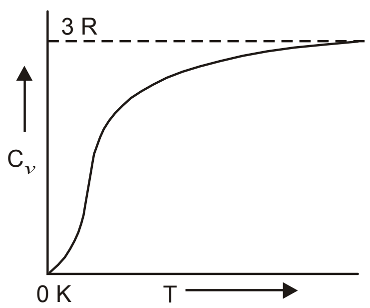


Fig. 4.5. Variation of C_v with T

4.8. VARIATION OF SPECIFIC HEAT OF WATER

Specific heat of water is the amount of heat energy required to raise the temperature of unit mass of water through 1°C or 1K.

Specific heat of water,

$$c = 1 \text{ cal g}^{-1}\,{}^\circ\text{C}^{-1} = 1 \text{ cal g}^{-1}\text{ K}^{-1}$$
$$= 4.2 \text{ J g}^{-1}\text{ K}^{-1} = 4200 \text{ J kg}^{-1}\text{ K}^{-1}$$

The specific heat of every liquid varies with temperature. However, water shows a peculiar variation. Considerable variations in the specific heat of water were first observed by Rowland.

The specific heat of water as a function of temperature from 0° to 100°C has been plotted in Fig. 4.6.

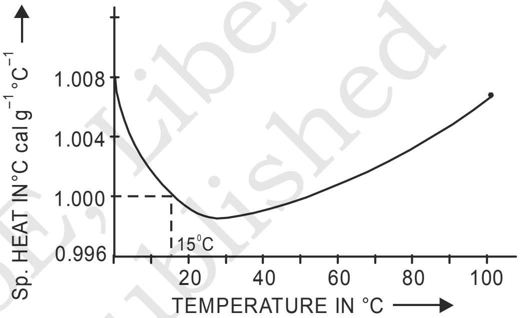


Fig. 4.6. Variation of specific heat of water with temperature

4.9. SPECIFIC HEATS OF A GAS

The ***specific heat*** *of a substance is the amount of heat required to increase the temperature of a unit mass of it through a unit temperature change.* Its unit in cgs system is cal $g^{-1}{}^\circ C^{-1}$. In mks system, it is measured in kilocal $kg^{-1}{}^\circ C^{-1}$. The SI unit is J kg^{-1} K^{-1}.

The above definition is based on the assumption that the heat supplied to the substance causes only a rise in the temperature of the substance. This assumption is valid only if the substance is heated at constant volume. But a substance generally expands when heated. In that case, the heat supplied is utilised in two ways. A part of the heat supplied is used in doing mechanical work in moving the molecules apart against forces of attraction between them and also in expanding against atmospheric pressure. The rest of the heat supplied increases the temperature of the substance.

In the case of solids and liquids, the coefficient is very small. So, the heat supplied is assumed to increase only the temperature. But the coefficient of expansion is very large in the case of gases. Thus, if a gas is heated, the heat energy will be required not only to increase the temperature of the gas but also to do mechanical work in overcoming external pressure during expansion. In the case of gases, only a negligible amount of mechanical work is required to pull the molecules apart because the intermolecular forces are extremely weak.

4.10. LIMITS OF SPECIFIC HEAT OF A GAS

Consider mass m of a gas. Let ΔQ units of heat raise the temperature of the gas through ΔT. Then the specific heat of the gas is given by

$$c = \frac{\Delta Q}{m\ \Delta T} \qquad [\because \ \Delta Q = mc\ \Delta T]$$

Consider a gas enclosed in a cylinder fitted with an air-tight and frictionless piston.

(i) Let the gas be suddenly compressed. In this case, no heat is supplied to the gas. But there is an increase in the temperature of the gas.

$$c = \frac{\Delta Q}{m\ \Delta T} = 0 \qquad [\because \ \Delta Q = 0]$$

(ii) Let the gas be heated and allowed to expand. Suppose the 'fall in temperature due to expansion' is equal to the 'rise in temperature due to heat supplied'.

$$\therefore \qquad c = \frac{\Delta Q}{m \times 0} = \infty \qquad [\because \ \Delta T = 0]$$

(iii) Let the gas be heated and allowed to expand. Suppose, in this case, the 'fall in temperature due to expansion' is less than the 'rise in temperature due to heat supplied'. The net effect will be a rise in the temperature of the gas. So, ΔT is positive. Thus $c = \left(\frac{\Delta Q}{m\ \Delta T}\right)$ is positive.

(iv) Let the gas be heated and allowed to expand such that the 'fall in temperature due to expansion' is more than the 'rise in temperature due to heat supplied'. The net effect will be a decrease in the temperature of the gas. So, ΔT is negative. Thus, $c = \left(\frac{\Delta Q}{m\ \Delta T}\right)$ is negative.

CONCLUSION

We can conclude from the above examples that a gas does not possess a unique or a single specific heat. The specific heat of a gas may have any positive or negative value ranging from zero to infinity. The specific heat of a gas depends upon the manner in which it is being heated. Thus, it is meaningless to talk about the specific heat of a gas unless the conditions under which it is being heated are mentioned.

4.11. HEAT TRANSFER

We have seen that heat is energy transfer from one system to another or from one part of a system to another part, arising due to temperature difference. What are the different ways by which this energy transfer takes place? There are three distinct modes of heat transfer: conduction, convection and radiation (Fig. 4.7).

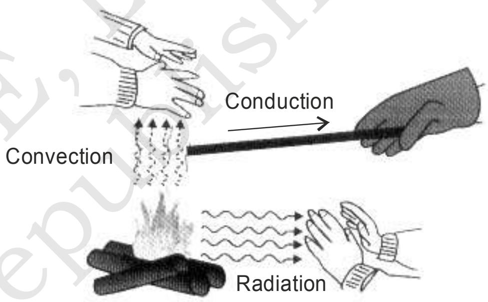


Fig. 4.7. Heating by conduction, convection and radiation

4.12. CONDUCTION

(*i*) *Conduction is the mechanism of transfer of heat between two adjacent parts of a body because of their temperature difference, without the actual movement of the particles from their equilibrium positions.*

Suppose one end of a metallic rod is put in a flame. The other end of the rod will soon feel so hot that you cannot hold it by your bare hands. Here heat transfer takes place by conduction from the hot end of the rod through its different parts to the other end.

The ability to conduct heat differs widely from substance to substance. Gases are poor thermal conductors. Liquids have conductivities intermediate between solids and gases.

(*ii*) Quantitative description of heat flow. Consider a metal rod of uniform cross-sectional area A with its two ends maintained at temperatures T_1 and T_2 such that $T_1 > T_2$. Let L be the length of the rod between the hot reservoir (at temperature T_1) and cold reservoir (at temperature T_2).

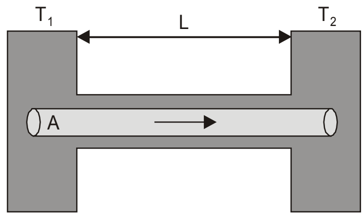


Fig. 4.8. Steady state heat flow by conduction in a bar with its two ends maintained at temperatures T_1 and T_2

It has been experimentally observed that in the steady state, the rate of flow of heat (or heat current) is

(*a*) directly proportional to the cross-sectional area A

$$H \propto A \qquad \ldots(1)$$

(*b*) directly proportional to the temperature difference $(T_1 - T_2)$ between the hot and cold faces

$$H \propto (T_1 - T_2) \qquad \ldots(2)$$

(*c*) inversely proportional to the distance L between the hot and cold reservoirs.

$$H \propto \frac{1}{L} \qquad \ldots(3)$$

Combining factors (1), (2) and (3), we get

$$H \propto \frac{A(T_1 - T_2)}{L}$$

or

$$\boxed{H = \frac{KA(T_1 - T_2)}{L}}$$

Here, K is a constant of proportionality called coefficient of thermal conductivity of the material of the block. Its value depends upon the nature of material of the block.

Total amount of heat Q flowing from hot to cold reservoir is the product of heat current H and time *t*.

$$\therefore \qquad Q = Ht$$

or

$$Q = \frac{KA(T_1 - T_2)t}{L} \qquad \ldots(4)$$

(*iii*) Coefficient of thermal conductivity. If A = 1, L = 1, t = 1 and $(T_1 - T_2) = 1$, then from equation (4), K = Q.

This leads us to the following definition of coefficient of thermal conductivity.

The coefficient of thermal conductivity of a material is defined as the quantity of heat flowing per second through a rod (or slab or block) of that material having unit length and unit cross-sectional area in the steady state when the difference of temperature between two ends of the rod (or slab or block) is 1°C or 1K and the flow of heat is perpendicular to the end-faces of the rod (or slab or block).

The coefficient of thermal conductivity may also be defined in terms of 'unit cube'.

The coefficient of thermal conductivity of a material is the quantity of heat flowing per second across the opposite faces of a unit cube, made of that material, when the opposite faces are maintained at a temperature difference of 1°C or 1K.

(*iv*) Concept of temperature gradient. *Temperature gradient is defined as the rate of change of temperature with distance between two isothermal surfaces.*

In equation (4), $\frac{T_1 - T_2}{L}$ gives the rate of change of temperature with distance. It is called temperature gradient. Let it be denoted by '$-\frac{dT}{dL}$'. Here negative sign indicates the decrease of temperature with distance.

Rewriting equation (4) in terms of temperature gradient, we get

$$Q = -KA\frac{dT}{dL}t$$

If $A = 1$, $-\frac{dT}{dL} = 1$ and $t = 1$, then $K = Q$.

*The **coefficient of thermal conductivity** of a material is the rate of flow of heat energy through a rod, made of that material, of unit cross-section area under a unit temperature gradient, the flow of heat being normal to the cross-sectional area.*

(*v*) Units of K. We know that

$$Q = \frac{KA(T_1 - T_2)t}{L} \quad \text{or} \quad K = \frac{QL}{A(T_1 - T_2)t}$$

In cgs system, Q, L, A, $(T_1 - T_2)$ and t are measured in cal, cm, cm^2, °C and s respectively. So, the cgs unit of K is cal $cm^{-1}{}^{\circ}C^{-1}\ s^{-1}$.

In SI, Q, L, A, $(T_1 - T_2)$ and t are measured in joule, m, m^2, K and s respectively.

So, the SI unit of K is J $m^{-1}\ K^{-1}\ s^{-1}$ or W $m^{-1}\ K^{-1}$.

(*vi*) Dimensional formula of K.

$$[K] = \frac{[Q]\ [L]}{[A]\ [T_1 - T_2]\ [t]}$$

$$= \frac{[ML^2T^{-2}]\ [L]}{[L^2]\ [K]\ [T]} = [ML\ T^{-3}\ K^{-1}]$$

(*vii*) Values of K. The thermal conductivities of various substances are listed in Table 4.1. These values vary slightly with temperature, but can be considered to be constant over a normal temperature range.

You may have noticed that some cooking pots have copper coating on the bottom. Being a good conductor of heat, copper promotes the distribution of heat over the bottom of a pot for uniform cooking. Plastic foams, on the other hand, are good insulators, mainly because they contain pockets of air. Recall that gases are poor conductors, and note the low thermal conductivity of air in the Table 4.1. Heat retention and transfer are important in many other applications. Houses made of concrete roofs get very hot during summer days, because thermal conductivity of concrete (though much smaller than that of a metal) is still not small enough. Therefore, people usually prefer to give

TABLE 4.1. Thermal Conductivities (K)

		$W\ m^{-1}\ K^{-1}$
Metals	Aluminium	205
	Brass	109
	Copper	385
	Lead	34.7
	Mercury	8.3
	Silver	406
	Steel	50.2
Non-metals	Body fat	0.20
	Insulating brick	0.15
	Red brick	0.6
	Concrete	0.8
	Cork	0.04
	Felt	0.04
	Glass	0.8
	Water	0.8
	Ice	1.6
	Glass wool	0.04
	Styrofoam	0.01
	Wood	0.12–0.04
Gases	Air	0.024
	Argon	0.016
	Helium	0.14
	Hydrogen	0.14
	Oxygen	0.023

a layer of earth or foam insulation on the ceiling so that heat transfer is prohibited and keeps the room cooler.

Generally, metals are good conductors of heat, silver being the best. Non-metals like wood, glass are poor conductors with low value of K.

4.13. APPLICATIONS OF THERMAL CONDUCTIVITY TO EVERYDAY LIFE

1. In winter, a brass knob appears colder than a wooden knob. It is due to the reason that the thermal conductivity of brass (K = 109 Wm^{-1} K^{-1}) is more than that of wood (K = 0.12 Wm^{-1} K^{-1}). When the brass knob is touched, the heat energy is quickly conducted away from the hand. On the other hand, when the wooden knob is touched, the flow of heat energy from the hand is extremely slow. Thus, a brass knob appears colder than a wooden knob although both may be at the same temperature.

2. Woollen clothes keep us warm. This is because wool contains air in its pores. Air (K = 0.024 Wm^{-1} K^{-1}) is a bad conductor of heat. In-fact, wool (K = 0.01 Wm^{-1} K^{-1}) is also a bad conductor of heat. Both the air and the wool do not permit heat to be conducted away from the body. So, the woollen clothes keep us warm.

3. A new quilt is warmer than old quilt. This is because new quilt contains more air in its pores as compared to old quilt. Since air is a bad conductor of heat therefore the heat is not conducted away from the body.

4. Cooking utensils are provided with wooden handles. This is because wood is a bad conductor of heat. So, the wooden handle would not permit heat to be conducted from hot utensil to hand. Thus, the hot cooking utensil can be easily held in hand through the wooden handle.

5. Cooking utensils are made of aluminium and brass. This is because aluminium and brass are good conductors of heat. They rapidly absorb heat from the fire and supply it to the food to be cooked.

6. Ice is covered in gunny bags to prevent melting of ice. This is because of the fact that gunny bags are bad conductors of heat. The pores of gunny bags contain air which is also a bad conductor of heat.

7. The double-walled houses of ice made by Eskimos are warm from inside. This is because the air within the walls does not allow heat to be conducted away to the outside air.

8. Two thin blankets are warmer than a single thick blanket. This is because the two thin blankets enclose a layer of air between them. Since air is a bad conductor of heat therefore the conduction of heat is prevented.

Example 2. *Two rods, one semi-circular and the other straight, of the same material and of same cross-sectional area are joined as shown in Fig. 4.9. The ends A and B are maintained at a constant temperature difference. Calculate the ratio of the heat conducted through a cross-section of a semi-circular rod to the heat conducted through a cross-section of the straight rod in a given time.*

Solution. We know that $Q = \dfrac{KA(T_1 - T_2)t}{L}$

In the given problem, $Q \propto \dfrac{1}{L}$

$$\therefore \qquad \frac{Q_1}{Q_2} = \frac{2r}{\pi r} = \mathbf{\frac{2}{\pi}}$$

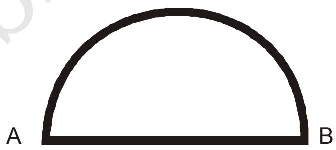


Fig. 4.9

Example 3. *One end of a 0.25 m long metal bar is in steam and the other in contact with ice. If 12 × 10^{-3} kg of ice melts per minute, what is the thermal conductivity of the metal? Given: cross-section of the bar = 5 × 10^{-4} m^2 and latent heat of ice is 80 kcal/kg.*

Solution. $L = 0.25$ m, $A = 5 \times 10^{-4}$ m^2,

$$(T_1 - T_2) = (100 - 0)\text{ K} = 100\text{ K}$$

$$t = 1 \text{ minute} = 60 \text{ s}$$

If Q is the amount of heat required to melt 12 × 10^{-3} kg of ice, then

$$Q = 12 \times 10^{-3} \times 80 \times 1000 \text{ cal}$$

$$Q = K\frac{A(T_1 - T_2)}{L}t \quad \text{or} \quad K = \frac{QL}{A(T_1 - T_2)t}$$

or

$$K = \frac{12 \times 10^{-3} \times 80 \times 1000 \times 0.25}{5 \times 10^{-4} \times 100 \times 60} \text{ cal s}^{-1}\text{ m}^{-1}\text{ K}^{-1}$$

$$= \mathbf{80 \text{ cal s}^{-1} \text{ m}^{-1} \text{ K}^{-1}}$$

4.14. CALORIMETRY

(*i*) Isolated System. *A system is said to be isolated if there is no transfer of heat between the system and its surroundings.* When different parts of an isolated system are at different temperatures, a quantity of heat transfers from the part at higher temperature to the part at lower temperature. The heat lost by the part at higher temperature is equal to the heat gained by the part at lower temperature.

(*ii*) Principle of Calorimetry. Calorimetry means measurement of heat. According to the principle of calorimetry, when a body at higher temperature is brought in contact with another body at lower temperature, the heat lost by the hot body is equal to the heat gained by the colder body, provided no heat is allowed to escape to the surroundings.

(*iii*) Calorimeter. *A device in which heat measurement can be made by utilising the principle of calorimetry is called a calorimeter.* It consists of a metallic vessel and stirrer of the same material like copper or aluminium. The vessel is kept inside a wooden jacket which contains heat insulating materials (glass, wool, etc.). The outer jacket acts as a heat shield and reduces the heat loss from the inner vessel. There is an opening in the outer jacket through which a mercury thermometer can be inserted into the calorimeter.

(*iv*) Thermal Capacity or Heat Capacity. *Thermal capacity of a body is the amount of heat required to raise the temperature of the body through 1°C or 1K.*

The amount of heat required to raise the temperature of a body of mass m through ΔT is,

$\Delta Q = ms\,\Delta T$, where s is the specific heat of the body under consideration.

If $\Delta T = 1$, then ΔQ = thermal capacity.

$\therefore$ Thermal capacity, $S = ms$ ...(1)

So, the thermal capacity of a body is the product of mass and specific heat of the body. The SI unit of thermal capacity is $J\,K^{-1}$.

(*v*) Water Equivalent. *Water equivalent of a body is defined as the mass of water in gram which absorbs or emits the same amount of heat as is done by the given body for the same rise or fall in temperature.*

It is denoted by w. Its SI unit is kg.

Now, $\Delta Q = w\,\Delta T = ms\,\Delta T$ or $w = ms$ ...(2)

It follows from equations (1) and (2) that the water equivalent and thermal capacity of a body are numerically equal. If water equivalent is measured in gram, the thermal capacity is measured in calorie/°C.

4.15. CHANGE OF STATE—LATENT HEAT CAPACITY

(*i*) Change of state. *A transition from one of the three states of matter to another is called a change of state.* Two common changes of states are solid to liquid and liquid to gas (and vice versa). These changes can occur when the exchange of heat takes place between the substance and its surroundings.

(*ii*) Melting. *The change of state from solid to liquid is called melting and from liquid to solid is called* ***fusion***. It is observed that the temperature remains constant until the entire amount of the solid substance melts. Both the solid and liquid states of the substance coexist in thermal equilibrium during the change of state from solid to liquid. **The temperature at which the solid and the liquid states of the substance coexist in thermal equilibrium with each other is called its melting point.** It is characteristic of the substance. It also depends on pressure. The melting point of a substance at standard atmospheric pressure is called its **normal melting point.**

After the whole of ice gets converted into water and as we continue further heating, we shall see that temperature begins to rise. The temperature keeps on rising till it reaches nearly 100°C when it again becomes steady. The heat supplied is now being utilised to change water from liquid state to vapour or gaseous state.

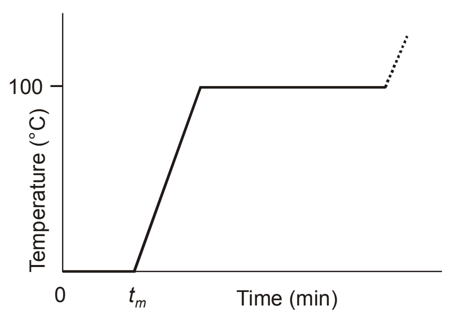


Fig. 4.10. A plot of temperature versus time showing the changes in the state of ice on heating (not to scale)

(*iii*) Vaporisation. *The change of state from liquid to vapour (or gas) is called vaporisation.* It is observed that the temperature remains constant until the entire amount of the liquid is converted into vapour. Both the liquid and vapour states of the substance coexist in thermal equilibrium, during the change of state from liquid to vapour. The temperature at which the liquid and the vapour states of the substance coexist is called its **boiling point.**

Boiling point decreases with decrease in pressure. This explains why cooking is difficult on hills. At high altitudes, atmospheric pressure is lower, reducing the boiling point of water as compared to that at sea-level. On the other hand, boiling point is increased inside a pressure cooker by increasing the pressure. Hence, cooking is faster. The boiling point of a substance at standard atmospheric pressure is called its **normal boiling point.**

(*iv*) Sublimation. All substances do not pass through the three states : solid-liquid-gas. There are certain substances which normally pass from the solid to the vapour state directly and vice versa. *The change from solid state to vapour state without passing through the liquid state is called* ***sublimation.*** The substance is said to sublime. Dry ice (solid CO_2) and iodine sublime. During the sublimation process, both the solid and vapour states of a substance coexist in thermal equilibrium.

(*v*) Latent heat. *The amount of heat per unit mass transferred during a change of state of the substance is called latent heat of transformation for the process.*

The heat required during a change of state depends upon the heat of transformation and the mass of the substance which completely undergoes a state change. Thus, if mass m of a substance undergoes a change from one state to the other completely, then the quantity of heat required is given by

$$Q = mL \quad \text{or} \quad L = Q/m$$

where L is known as latent heat. It is a characteristic of the substance. Its SI unit is J kg^{-1}. The value of L depends on the pressure at which it is measured. Its value at standard atmospheric pressure is usually quoted. The latent heat for a solid-liquid state change is called the **latent heat of fusion (L_f),** and that for a liquid-gas state change is called the **latent heat of vaporisation (L_v).** These are often referred to as simply the heat of fusion and the heat of vaporisation (v_p).

The variation of temperature with heat energy for a given quantity of water is shown in Fig. 4.11.

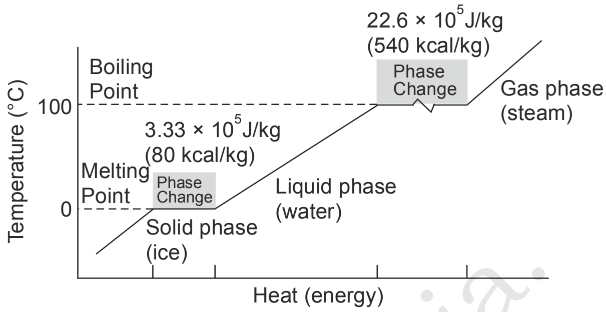


Fig. 4.11. Temperature versus heat for water (Not to scale)

The following facts are clear from the graph:

(i) When heat is added (or removed) during a change of state, the temperature remains constant.

(ii) The slopes of the phase lines are not all the same. This indicates that specific heats of the various states are not equal.

(iii) For water, the latent heats of fusion and vaporisation are $L_f = 3.33 \times 10^5$ J kg^{-1} and $L_v = 22.6 \times 10^5$ J kg^{-1} respectively. So, 3.33×10^5 J of heat is needed to melt 1 kg of ice at 0°C, and 22.6×10^5 J of heat is needed to convert 1 kg of water to steam at 100°C. So, steam at 100°C carries 540 cal/g more heat than water at 100°C. This is why burns from steam are usually more serious than those from boiling water.

Example 4. *When 0.15 kg of ice of 0°C mixed with 0.30 kg of water at 50°C in a container, the resulting temperature is 6.7°C. Calculate the heat of fusion of ice. (s_{water} = 4186 J kg^{-1} °C^{-1}).*

Solution. Heat lost by water

= (0.30 kg) (4186 J kg^{-1}°C^{-1}) (50.0°C – 6.7°C)

= 54376.14 J

Heat to melt ice = (0.15 kg) L_f

Heat to raise temperature of ice water to final temperature

= (0.15 kg) (4186 J kg^{-1}°C^{-1}) (6.7°C – 0°C)

= 4206.93 J

Heat lost = heat gained

54376.14 = (0.15 kg) L_f + 4206.93 J

L_f = **3.34×10^5 J kg^{-1}**

Example 5. *How much heat energy is liberated when 100 g of copper in a vessel is cooled from 100°C to 50°C ? Given : specific heat capacity copper,* s_{Cu} *= 385 J kg*$^{-1}$*°C*$^{-1}$*.*

Solution. Given, mass of the copper,

$$m = 100 \text{ g} = 0.1 \text{ kg}$$

Initial temperature of copper, $T_i = 100°C$,

Final temperature of copper, $T_f = 50°C$,

and specific heat capacity of copper, $s_{Cu} = 385 \text{ J kg}^{-1}°C^{-1}$.

Thus, the quantity of heat energy exchanged is given by

$$\Delta Q = m\, s_{Cu}\, \Delta T$$

$$= (0.1 \text{ kg})(385 \text{ J kg}^{-1}°C^{-1}) \times (50°C - 100°C)$$

i.e.,

$$\Delta Q = (0.1 \text{ kg})(385 \text{ J kg}^{-1}°C^{-1})(-50°C)$$

or

$$\Delta Q = \mathbf{-1925 \text{ J}}$$

The negative sign shows that heat is liberated.

4.16. CONVECTION

Convection *is the process in which heat is transferred from one point to another by the actual motion of matter from a region of high temperature to a region of lower temperature.*

Convection is possible only in fluids. Convection can be natural or forced. Natural convection is responsible for many familiar phenomena. In natural convection, gravity plays an important part. When a fluid is heated from below, the hot part expands and, therefore, becomes less dense. Because of buoyancy, it rises and the upper colder part replaces it. This again gets heated, rises up and is replaced by the colder part of the fluid. The process goes on. This mode of heat transfer is evidently different from conduction. Convection involves bulk transport of different parts of the fluid. In forced convection, material is forced to move by a pump or by some other physical means. The common examples of forced convection systems are forced-air heating systems in home, the human circulatory system, and the cooling system of an automobile engine. In the human body, the heart acts as the pump that circulates blood through different parts of the body, transferring heat by forced convection and maintaining it at a uniform temperature.

If the material is forced to move by a blower or pump, the process is called ***forced convection.*** *If the material flows due to difference in density (caused by thermal expansion), the process is called* ***natural*** *or* ***free convection.***

4.17. PHENOMENA BASED ON CONVECTION

(*i*) Land and Sea Breezes

It is an example of natural convection.

During the day, the ground heats up more quickly than large bodies of water do. This occurs both because the water has a greater specific heat and because mixing currents disperse the absorbed heat throughout the great volume of water. The air in contact with the warm ground is heated by conduction. It expands, becoming less dense than the surrounding cooler air. As a result, the warm air rises (air currents) and other air moves (winds) to fill the space-creating a **sea breeze** near a large body of water. Cooler air descends, and a thermal convection cycle is set up, which transfers heat away from the land. At night, the ground loses its heat more quickly, and the water surface is warmer than the land. As a result, the cycle is reversed and **land breeze** is there (Fig. 4.12).

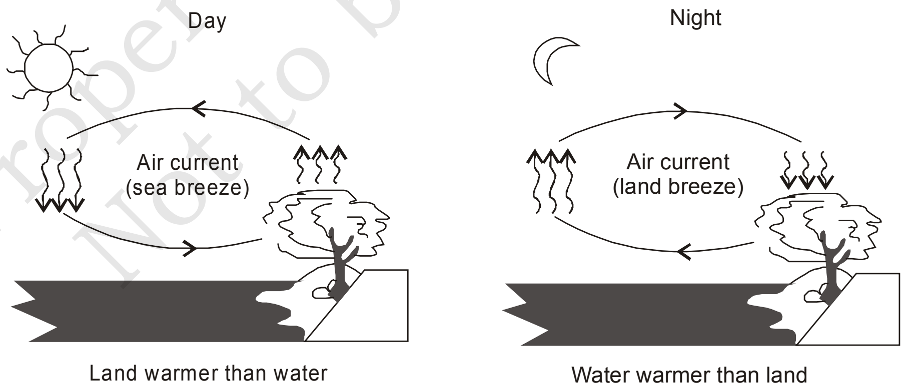


Fig. 4.12. Convection cycles

(*ii*) Formation of Trade Winds

The steady wind blowing from north-east to equator, near the surface of Earth, is called ***trade wind.*** It is an example of natural convection.

The equatorial and polar regions of the Earth receive unequal solar heat. Air at the Earth's surface near the equator is hot while the air in the upper atmosphere of the poles is cool. In the absence of any other factor, a convection current would be set up, with the air at the equatorial surface rising and moving out towards the poles, descending and streaming in towards the equator. The rotation of the Earth, however modifies this convection current.

(*iii*) To Regulate Temperature in the Human Body

Heat transfer in the human body involves a combination of mechanisms. These together maintain a remarkably uniform temperature in the human body inspite of large changes in environmental conditions.

The chief internal mechanism is forced convection. The heart serves as the pump and the blood as the circulating fluid.

4.18. RADIATION

The transfer of heat from one place to another in a straight line, with the speed of light, without heating the intervening medium is called radiation.

Conduction and convection require some material as a transport medium. These modes of heat transfer cannot operate between bodies separated by a certain distance in vacuum. The third mechanism for heat transfer needs no medium; it is called radiation. In radiation, the heat flows by means of electromagnetic waves. The energy so radiated in the form of electromagnetic waves is called radiant energy.

Electromagnetic waves do not require a material medium for propagation. These waves travel with a speed of 3×10^8 m s^{-1} in vacuum. This explains as to how heat transfer by radiation does not need any medium and why it is so fast. Heat from the Sun reaches the Earth by radiation. Similarly, we feel the warmth of nearby fire due to radiation.

4.19. NATURE OF THERMAL RADIATION

The energy emitted by a body in the form of radiation by virtue of its temperature is called ***thermal radiation.*** This energy is emitted by all bodies above absolute zero and is also called radiant energy. The most powerful source of radiant energy is the Sun.

Thermal radiation belongs to the electromagnetic family *i.e.,* it resembles γ-rays, X-rays, ultraviolet light, visible light and radiowaves. It can travel through vacuum and other transparent media. Its speed is the same as that of light. Like other electromagnetic radiations, it exhibits the phenomena of reflection, refraction, interference, diffraction and polarisation.

The wavelength of thermal radiation is longer than that of visible light. The wavelength of thermal radiation ranges from 8×10^{-7} m to 3×10^{-4} m whereas the wavelength of visible light ranges from 4×10^{-7} m to 8×10^{-7} m.

4.20. PROPERTIES OF THERMAL RADIATION

Thermal radiation has got following properties:

(*i*) Thermal radiation can travel through vacuum.

(*ii*) Thermal radiation travels in straight lines.

(*iii*) Thermal radiation travels equally, in all directions, in a homogeneous medium.

(*iv*) Thermal radiation travels with the speed of light.

(*v*) Thermal radiation does not heat the medium through which it passes.

(*vi*) Thermal radiation obeys inverse square law. The intensity of radiation at a point is inversely proportional to the square of the distance of the point from the source of radiation.

(*vii*) Thermal radiation obeys the laws of reflection.

(*viii*) Thermal radiation can be refracted.

4.21. BLACK BODY

A perfectly black body is one which absorbs completely the radiations of all wavelengths falling on it. Since a perfectly black body neither reflects nor transmits any radiation therefore its absorptance is unity. It is for the same reason that it appears black irrespective of the wavelength of incident radiation.

A perfectly black body cannot be *realised in practice. The nearest approach to a perfect black body* is a surface coated with lamp black or platinum black. Such a surface absorbs 96% to 98% of the incident radiation.

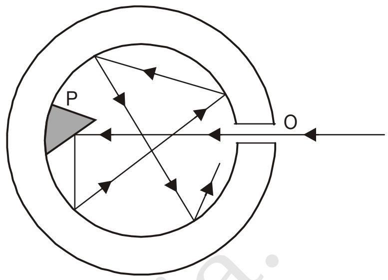


Fig. 4.13. Black body

For accurate experimental work, the black body designed by Fery is generally used. **Fery black body** is a closed double-walled hollow sphere having small opening O and a conical projection P opposite to the opening. The projection will protect direct reflection of any radiation in the opening from the surface opposite it. It is painted black from inside. Radiation entering the opening O suffers multiple reflections at the inner walls. After a few reflections, almost the entire radiation gets absorbed. *As an example, let 80% of energy be reflected at each reflection, the remaining 20% being absorbed.* Then, at two reflections, 64% will be reflected and 36% will be absorbed. Thus, nearly 99% of the energy will be absorbed in 10 reflections.

When the body is heated, it becomes a source of thermal radiation. The radiation from a constant temperature enclosure depends only on the temperature of the enclosure. It does not depend on the nature of the substance of which the enclosure is made.

4.22. STEFAN'S LAW OF BLACK BODY RADIATION

Statement. ***The total amount of heat energy radiated per second per unit area of a perfect black body is directly proportional to the fourth power of the absolute temperature of the surface of the body.***

This law is also known as Stefan's fourth power law.

If E be the *energy radiated by a unit area of the surface of black body per second* at absolute temperature T, then

$$E \propto T^4 \quad \text{or} \quad E = \sigma T^4$$

where σ is a constant known as Stefan's constant. Its value in SI units is

$$5.67 \times 10^{-8}\ \text{J m}^{-2}\ \text{s}^{-1}\ \text{K}^{-4} \qquad \text{or} \qquad \text{W m}^{-2}\ \text{K}^{-4}$$

Stefan derived this law experimentally in 1879. In 1884, Boltzmann gave a theoretical proof of this law based on thermodynamical considerations. So, this law is also known as **Stefan-Boltzmann law.**

It may be pointed out that the above law is not a law of cooling. It does not refer to the net loss of heat by a body. *It merely deals with the amount of heat energy radiated by the body by virtue of its temperature,* irrespective of what it gains from the surroundings. Moreover, Stefan's law applies to the whole range of wavelengths, without being limited to any particular wavelength.

Stefan's law can be extended to represent the *net loss of heat* by a body.

Consider a black body at absolute temperature T sorrounded by another black body at absolute temperature T_0. A unit area of the 'inner' black body loses heat energy σT^4 per second. But it also gains heat energy σT_0^4 per second.

$\therefore$ Net loss of heat energy per unit area per unit time,

$$E = \sigma T^4 - \sigma T_0^4 = \sigma (T^4 - T_0^4)$$

If the body is not a perfect black body, then $E = e\sigma (T^4 - T_0^4)$ where e is called the radiation emissivity or emissivity of the surface.

The value of e depends upon the nature of the surface.

4.23. DEDUCTION OF NEWTON'S LAW OF COOLING FROM STEFAN'S LAW

$$\begin{aligned} E &= \sigma (T^4 - T_0^4) \\ &= \sigma (T^2 - T_0^2)(T^2 + T_0^2) \\ &= \sigma (T - T_0)(T + T_0)(T^2 + T_0^2) \end{aligned}$$

If T is nearly equal to T_0, then

$$\begin{aligned} E &= \sigma (T - T_0)(2T_0)(2T_0^2) \\ &= \sigma (T - T_0)(4T_0^3) = 4\sigma T_0^3 (T - T_0) \end{aligned}$$

If A be the total surface area, then loss of heat energy per unit time or rate of loss of heat

$$= 4 A\sigma T_0^3 (T - T_0).$$

$\therefore$ rate of loss of heat $\propto (T - T_0)$

So, ***the rate of loss of heat is proportional to the difference of temperature between the body and surroundings.*** This is Newton's law of cooling.

4.24. GREENHOUSE EFFECT

We can see the Sun and the stars clearly through the atmosphere because the atmosphere is transparent to visible radiation. However, most of the infrared radiation is absorbed by the atmosphere. So, most of the infrared radiation cannot pass through the atmosphere. Now, the energy from sunlight obviously heats the Earth, which like any other hot body, starts emitting radiation. However, the Earth is much cooler than the Sun so that, according to Planck's law, its radiation is mostly in the infrared region. This is unlike the solar radiation. The radiation from the Earth is unable to cross the lower atmosphere which reflects it right back. Thus, **the Earth's atmosphere is richer in infrared radiaton** which is sometimes called "heat radiation". This is because most materials absorb these radiations quickly 'heating' themselves up in the process. The clouds that are low also prevent infrared radiation from passing through. This helps to keep the Earth's surface warm at night. This phenomenon is popularly known as **"Greenhouse effect".**

Fig. 4.14 shows a Greenhouse effect. *The surface of the Earth absorbs solar radiation which passes through the atmosphere. In turn, the Earth radiates infrared waves. These waves are reflected back by clouds and gases in the lower atmosphere.*

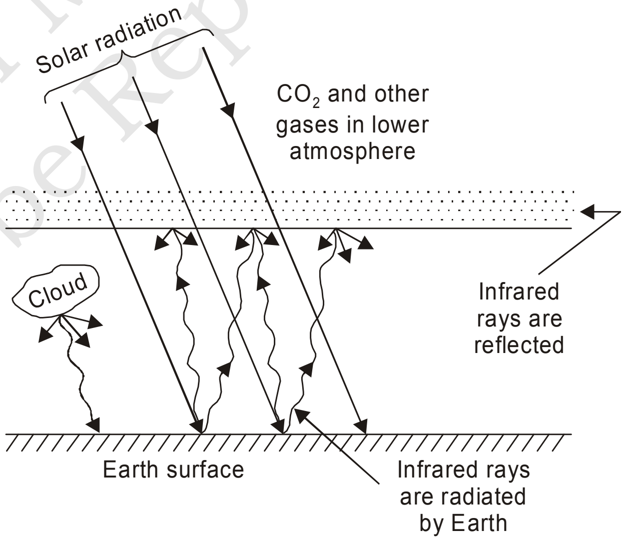


Fig. 4.14. Greenhouse effect

The components of solar and other extra-terrestrial sources in the ultraviolet and lower wavelength domains are dangerous. This is because they cause genetic damages to living cells. It is the ozone layer which blocks the passage of UV radiation and protects us from the harmful portions of solar radiation. Nearly all radiation of wavelength less than 3×10^{-7} m is absorbed by the ozone layer.

The Earth may be regarded as a huge Greenhouse where the glass of the artificial Greenhouse is replaced by atmosphere. The carbon dioxide and the water vapours present in the Earth's atmosphere are good absorbers of infrared radiation.

REVIEW EXERCISES

Do the review exercises in your notebook.

A. Multiple Choice Questions

1. A metal cube of length 10.0 mm at 0°C (273 K) is heated to 200°C (473 K). Given : its coefficient of linear expansion is 2×10^{-5} K^{-1}. The per cent change of its volume is

(*a*) 0.1 (*b*) 0.2

(*c*) 0.4 (*d*) 1.2.

2. Which of following quantities must be determined so that the thermal capacity of a body may be calculated, when the specific heat of body is known?

(*a*) Emissivity (*b*) Latent heat

(*c*) Mass (*d*) Temperature.

3. The density of water at 20°C is 998 kg m^{-3} and at 40°C, it is 992 kg m^{-3}. The coefficient of cubical expansion of water is nearest to

(*a*) 2×10^{-4}/°C (*b*) 4×10^{-4}/°C

(*c*) 6×10^{-4}/°C (*d*) 3×10^{-4}/°C.

4. The density of water at 4°C is 1000.0 kg/m^3 and at 100°C it is 958.4 kg/m^3. The cubic expansivity of water between these temperatures is

(*a*) 4.5×10^{-3} K^{-1} (*b*) 5.4×10^{-5} K^{-1}

(*c*) 4.5×10^{-4} K^{-1} (*d*) 5.4×10^{-6} K^{-1}.

5. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then

(*a*) $C_p - C_v = 28R$ (*b*) $C_p - C_v = R/14$

(*c*) $C_p - C_v = R/28$ (*d*) $C_p - C_v = R$.

6. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(*a*) $\frac{1}{T}$ (*b*) $\frac{2}{T}$

(*c*) $\frac{3}{T}$ (*d*) $\frac{4}{T}$.

7. Two uniform brass rods A and B of lengths l and $2l$ and radii $2r$ and r respectively are heated to the same temperature. The ratio of the increase in the volume of A to that of B is

(*a*) 1 : 1 (*b*) 1 : 2

(*c*) 2 : 1 (*d*) 1 : 4.

8. 0.1 m^3 of water at 80°C is mixed with 0.3 m^3 of water at 60°C. The final temperature of the mixture is

(*a*) 65°C (*b*) 70°C

(*c*) 60°C (*d*) 75°C.

9. The resistance of the wire in the platinum resistance thermometer at ice point is 5 Ω and at steam point is 5.25 Ω. When the thermometer is inserted in an unknown hot bath, its resistance is found to be 5.5 Ω. The temperature of the hot bath is

(*a*) 100°C (*b*) 200°C

(*c*) 300°C (*d*) 350°C.

10. 10 mole of an ideal monatomic gas at 10°C is mixed with 20 mole of another monatomic gas at 20°C. Then the temperature of the mixture is

(*a*) 15.5°C (*b*) 15°C

(*c*) 16°C (*d*) 16.6°C.

B. Fill in the Blanks

1. A piece of ice (heat capacity = 2100 J kg^{-1} $°C^{-1}$ and latent heat = 3.36×10^5 J kg^{-1}) of mass m gram is at – 5°C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice-water mixture is in equilibrium, it is found that 1 g of ice has melted. Assuming there is no other heat exchange in the process, the value of m is __________ .

2. Certain amount of heat is given to 100 g of copper to increase its temperature by 21°C. If the same amount of heat is given to 50 g of water, then the rise in its temperature is __________ .

3. A thin copper rod rotates about an axis passing through its end and perpendicular to its length with an angular speed ω_0. The temperature of the copper rod is increased by 100°C. If the coefficient of linear expansion of copper is 2×10^{-5}/°C, the percentage change in the angular speed of the rod is __________ .

4. A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by t°C, the linear stress developed in it is __________ .
5. An aluminium sphere of 20 cm diameter is heated from 0°C to 100°C. Its volume changes by __________ . (Given that coefficient of linear expansion for aluminium $\alpha_{Al} = 23 \times 10^{-6}$/°C)
6. A lead bullet strikes against a steel plate with a velocity 200 m s^{-1}. If the impact is perfectly inelastic and the heat produced is equally shared between the bullet and the target, then the rise in temperature of the bullet is __________ .
7. Two temperature scales A and B are related by $\frac{A-42}{110} = \frac{B-72}{220}$. At __________ temperature two scales have the same reading?
8. When the temperature of a rod increases from t to $t + \Delta t$, its moment of inertia increases from I to I + ΔI. If α be the coefficient of linear expansion of the rod, then the value of $\frac{\Delta I}{I}$ is __________ .
9. If an anisotropic solid has coefficients of linear expansion α_x, α_y and α_z for three mutually perpendicular directions in the solid, its coefficient of volume expansion will be __________ .
10. If the pressure and the volume of certain quantity of ideal gas are halved, then its temperature __________ .

C. Very Short Answer Questions

1. Is the bulb of a thermometer made of diathermic or adiabatic wall?
2. Why does a metal bar appear hotter than a wooden bar at the same temperature? Equivalently it also appears cooler than wooden bar if they are both colder than room temperature.
3. Calculate the temperature which has same numeral value on Celsius and Fahrenheit scale.
4. These days people use steel utensils with copper bottom. This is supposed to be good for uniform heating of food. Explain this effect using the fact that copper is the better conductor.
5. Calculate the stress developed inside a tooth cavity filled with copper when hot tea at temperature of 57°C is drunk. You can take body (tooth) temperature to be 37°C and $\alpha = 1.7 \times 10^{-5}$/°C, bulk modulus for copper = 140×10^{9} N m^{-2}.

D. Short Answer Questions

1. Find out the increase in moment of inertia I of a uniform rod (coefficient of linear expansion α) about its perpendicular bisector when its temperature is slightly increased by ΔT.
2. During summers in India, one of the common practice to keep cool is to make ice balls of crushed ice, dip it in flavoured sugar syrup and sip it. For this a stick is inserted into crushed ice and is squeezed in the palm to make it into the ball. Equivalently in winter in those areas where it snows, people make snow balls and throw around. Explain the formation of ball out of crushed ice or snow in the light of P-T diagram of water.
3. 100 g of water is supercooled to – 10°C. At this point, due to some disturbance mechanised or otherwise some of it suddenly freezes to ice. What will be the temperature of the resultant mixture and how much mass would freeze?
   [S_w = 1 cal/g/°C and L^{w}_{Fusion} = 80 cal/g]
4. One day in the morning, Ramesh filled up 1/3 bucket of hot water from geyser, to take bath. Remaining 2/3 was to be filled by cold water (at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh had to attend to something which would take some time, say 5 – 10 minutes before he could take bath. Now he had two options: (*i*) fill the remaining bucket completely by cold water and then attend to the work, (*ii*) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer? Explain.
5. A thin rod having length L_0 at 0°C and coefficient of linear expansion a has its two ends maintained at temperatures θ_1 and θ_2, respectively. Find its new length.

E. Long Answer Questions

1. A geyser heats water flowing at the rate of 3.0 litre per minute from 27°C to 77°C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel, if its heat of combustion is 4.0×10^4 J g^{-1} ?
2. Calculate the difference between two specific heats of 1 g of helium gas at NTP. Molecular weight of helium = 4 and J = 4.186×10^7 erg cal^{-1}.
3. How much heat energy is absorbed when 50 g ice cube melts at 0°C ? (Latent heat of fusion of ice, $L_f = 3.35 \times 10^5$ J kg^{-1})
4. A small hole is made in a hollow sphere whose walls are at 723°C. Find the total energy radiated per second per cm^2.
5. Calculate the temperature in kelvin at which a perfect black body radiates at the rate of 5.67 W cm^{-2}. Stefan's constant is 5.67×10^{-5} erg s^{-1} cm^{-2} K^{-1}.

SEMESTER-2 (Period-V)

TOPIC 5

Waves

P11CH5

5.1. INTRODUCTION

Energy can be transferred from one place to another through the bulk motion of matter. A running stream of water carries energy with itself as it moves along. There is another way of transferring energy in which there is no bulk motion of matter. This is by means of 'waves'. The waves are of three types—mechanical waves, electromagnetic waves and matter waves.

(*i*) **Mechanical waves** can be produced and propagated only in those material media which possess elasticity and inertia. These waves are also called elastic waves. Common examples include water waves, sound waves, and seismic waves. They can exist only within a material medium such as water, air and rock.

(*ii*) **Electromagnetic waves** do not require any material medium for their production or propagation. Common examples include visible and ultraviolet light, radio and television waves, microwaves, X-rays and radio waves. All electromagnetic waves travel through vacuum at the same speed c given by

$$c = 299792458 \text{ m s}^{-1} \approx 3 \times 10^8 \text{ m s}^{-1}$$

(*iii*) **Matter waves** are waves associated with electrons, protons and other fundamental particles, and even atoms and molecules.

In this unit, we shall study only mechanical wave motion which will be referred to simply as wave motion.

5.2. WAVES ON SURFACE OF WATER

We can see and appreciate waves on a sea-shore. Waves can be generated in a large basin or a tub of water by just dropping a small stone or a pebble at the centre.

The dropped pebble creates a disturbance in the centre. The particles of water acquire energy (both kinetic and potential). This energy is transmitted to the next portion of the surface layer and so on. Thus we see something travelling outwards away from the source of disturbance in ever-expanding concentric circles (Fig. 5.1). In some regions, water level is below the usual normal level. These are called *troughs.* On either side of a trough, there are regions where water is at a level higher than the normal. These are called *crests.*

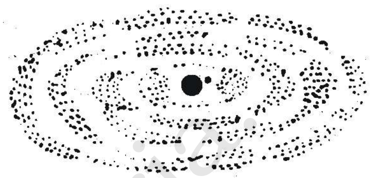


Fig. 5.1. Water waves

To sum up, *the disturbance moves progressively onwards in the form of alternate troughs and crests as shown in Fig. 5.2.* This disturbance is called 'wave'.

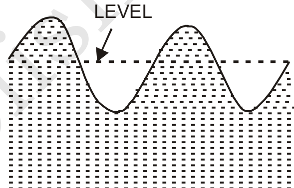


Fig. 5.2. Crests and troughs

It is the disturbance which travels outwards and not water. This fact can be verified by placing a piece of cork or a straw on the disturbed surface of water. It will be observed that the cork or straw just keeps on oscillating up and down about its mean position, sometimes riding a crest and at another time resting on a trough. The straw or cork will not move outwards with the disturbance.

Thus, the particles of the medium certainly oscillate about their mean positions but their permanent physical movement away from their original positions is not there.

Wave motion *may be defined as a form of disturbance which is due to the repeated periodic vibrations of the particles of the medium about their mean positions and the motion is handed over from one particle to the other without any net transport of the medium.*

It may also be defined as under:

Wave motion *is a means of transferring momentum and energy from one point to another without any transport of matter between the two points.*

5.3. WAVES IN STRINGS

Consider a stretched string tied at one end to a fixed support. Let the free end of the stretched string be given an upward jerk. This will produce an upward kink in the string. This upward kink travels, along the string, towards the fixed end as shown in Fig. 5.3.

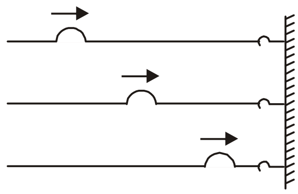


Fig. 5.3. Waves in strings

It may be noted that it is only the disturbance given to the free end that travels along the string and not any part of the string itself.

If the free end of the string is given one complete oscillation, then an upward kink will be followed by a downward kink along the string. However, if we continuously move the free end of the string up and down, a wave-train is observed to move, along the string, having alternate crests and troughs (Fig. 5.4).

Fig. 5.4. Wave-train in string

5.4. CHARACTERISTICS OF WAVE MOTION

(*i*) Wave motion is merely *a form of disturbance which is produced in the medium by the repeated periodic motion of the particles of the medium about their mean positions.*

(*ii*) The energy moves outwards away from the source while the particles of the medium continue vibrating about their mean positions with fixed frequency. Thus, a wave represents the transfer of energy from particle to particle. Energy can be transmitted over long distances by wave motion.

(*iii*) In order to set up wave motion in a medium, it is necessary that the medium should possess elasticity and inertia. Due to elasticity, the medium has a tendency to come back to its original condition. Due to inertia, the medium can store energy. The speed of a wave in a medium is determined by the inertia and elasticity of the medium. So, material media (having elasticity and inertia) are capable of transmitting mechanical waves. On the other hand, no material medium is necessary for the propagation of electromagnetic waves.

(*iv*) During their to and fro vibration about their mean positions, the particles possess different velocities. At the extreme position, the particle velocity is zero. The velocity increases as the particle moves towards the mean position. At the mean position, the particle velocity is maximum. The 'maximum velocity' is determined by the energy of the wave. On the other hand, a wave propagates with constant velocity in a homogeneous and isotropic medium.

To sum up, the wave velocity is very much different from the particle velocity.

(v) Depending upon the type of wave, the particles of the medium may actually oscillate up and down or the particles may move towards or against the direction of propagation of wave.

(vi) In a wave motion, all the particles of the medium do not start moving at once. But there is a constant phase difference between one particle and the next. The wave advances in that direction in which it meets particles with continuously decreasing phase. In simple words, the movement of each particle begins a little later than that of its predecessor.

5.5. TYPES OF MECHANICAL WAVES

Mechanical waves can be divided into two types :

(*i*) Transverse waves (*ii*) Longitudinal waves.

5.5.1. Transverse Wave Motion

Transverse wave motion *is that wave motion in which the individual particles of the medium execute simple harmonic motion about their mean positions in a direction perpendicular to the direction of propagation of the wave.* The wave itself is known as transverse wave.

The water waves, the movement of a kink in a rubber string, the movement of string in a '*sitar*' or a violin, the movement of the membrane of a 'tabla' or 'dholak' are all **examples** of transverse vibrations of these media and **transverse waves** generated in those media.

A transverse wave progresses as a series of troughs and crests. ***Crest*** *is the position of maximum displacement in the positive direction i.e., above the line of mean position or normal level.* As an example, in Fig. 5.5, A, C and E are crests. *When the displacement of a particle is maximum above the line of mean position, the particle is said to be at the crest of a wave.*

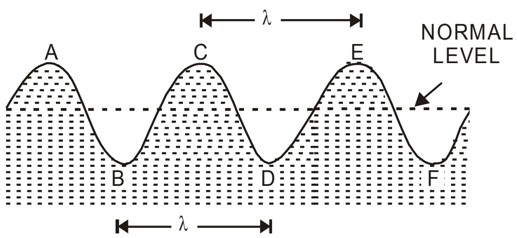


Fig. 5.5. Transverse wave

Trough *is the position of maximum displacement in the negative direction, i.e., below the line of mean position or normal level.* In Fig. 5.5, B, D and F are troughs. *When the displacement of a particle is maximum below the line of mean position, the particle is said to be at the trough of a wave.*

The distance between two consecutive crests or troughs is known as the wavelength.

Transverse waves can be transmitted through solids. They can also be set up on the surfaces of liquids. These waves cannot be transmitted inside liquids and gases. This is due to the fact that liquids and gases do not possess internal transverse restoring forces.

5.5.2. Longitudinal Wave Motion

Longitudinal wave motion *is that wave motion in which the individual particles of the medium execute simple harmonic motion about their mean positions along the direction of propagation of the wave.*

Sound wave is an example of longitudinal wave.

When a longitudinal wave travels through a medium, it produces compressions and rarefactions of the medium.

In a compression, the distance between any two consecutive particles of the medium is less than the normal distance.

So, the density of the medium in compression is more than the normal density.

In a rarefaction, the distance between any two consecutive particles of the medium is more than the normal distance. So, the density of the medium in a rarefaction is less than the normal density.

In Fig. 5.6 (*i*), the positions of different layers of air are shown when the tuning fork is not vibrating. However, when the tuning fork is set into vibration, the vibrating tuning fork sends out alternate waves of compression (or condensation) and rarefaction as depicted in Fig. 5.6 (*ii*). When these waves strike the ear drum of the listener, they make the ear drum

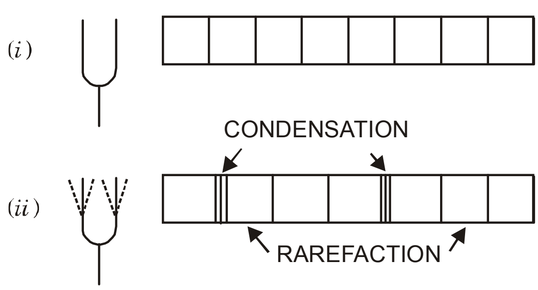


Fig. 5.6. Positions of different layers of air when (*i*) tuning fork is not vibrating (*ii*) tuning fork is vibrating

vibrate with the frequency of the incident waves. The distance between the centres of two nearest condensations or rarefactions is known as wavelength λ.

Again, consider the case of a spiral spring. When it is compressed at one end and released, the coils of the spring vibrate about their original positions along the length of the spring (Fig. 5.7). It will be observed that coils get closer together and move farther apart alternately. (AB + BC), *i.e.,* a compression and an adjoining rarefaction constitute one wave. Similarly, (BC + CD) or (CD + DE) or (DE + EF) constitute one wave.

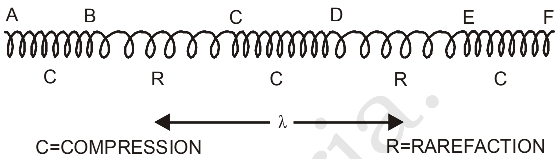


Fig. 5.7. Formation of compressions and rarefactions in spring

The succession of waves constitutes a wave train ABCDEF.

The longitudinal wave can be transmitted through solids, liquids or gases. In-fact, longitudinal wave is the only type of wave which can be propagated by a gas.

5.6. DISTINCTION BETWEEN LONGITUDINAL AND TRANSVERSE WAVES

Longitudinal Waves	Transverse Waves
1. The particles of the medium vibrate along the direction of propagation of the wave.	1. The particles of the medium vibrate at right angles to the direction of propagation of the wave.
2. The longitudinal waves travel in the form of alternate compressions (condensations) and rarefactions. One compression and one rarefaction constitute one wave.	2. The transverse waves travel in the form of alternate crests and troughs. One crest and one trough constitute one wave.
3. These waves can be formed in anymedium (solid, liquid or gas).	3. These waves can be formed in solids and on the surfaces of liquids only.
4. When longitudinal waves propagate, there are pressure changes in the medium.	4. When transverse waves propagate, there are no pressure changes in the medium.

5.7. IMPORTANT TERMS USED IN THE STUDY OF WAVE MOTION

(*i*) Crest. The elevation or hump caused in a medium due to the propagation of transverse wave through it is called crest.

(*ii*) Trough. The depression or hollow caused in a medium due to the propagation of transverse wave through it is called trough.

(*iii*) Compression. A portion of the medium where an increase in density occurs (because of reduction in volume) due to passage of longitudinal wave in it is called compression or condensation.

(*iv*) Rarefaction. A portion of the medium where a decrease in density occurs (because of increase in volume) due to passage of longitudinal wave in it is called rarefaction.

(*v*) Wavelength (λ). Following are the different ways of defining wavelength:

Wavelength of a wave is the distance travelled by the wave in a medium during the time a particle of the medium completes one vibration.

Wavelength is the distance between any two nearest particles of the medium vibrating in the same phase.

Wavelength is the distance between two consecutive crests or troughs.

Wavelength is the distance between two consecutive compressions or rarefactions.

(*vi*) Frequency (ν). Frequency of a wave is the number of complete wavelengths travelled by the wave in one second.

(*vii*) Time Period (T). Time period of a wave is the time taken by the wave to travel a distance equal to one wavelength.

5.8. RELATION BETWEEN FREQUENCY AND TIME PERIOD

Frequency of wave, ν = Frequency of vibration of the particles of the medium

Time period of wave, T = Time period of vibration of the particles of the medium

Time taken to complete ν vibrations is 1 second.

Time taken to complete 1 vibration is $\frac{1}{\nu}$ second. But this time is equal to time period T.

$$\therefore \qquad T = \frac{1}{\nu} \qquad \text{or} \qquad \nu = \frac{1}{T} \qquad \text{or} \qquad \nu\, T = 1$$

5.9. RELATION BETWEEN VELOCITY, FREQUENCY AND WAVELENGTH OF A WAVE

Distance travelled by wave in time T = λ

Distance travelled by wave in unit time = $\frac{\lambda}{T}$

$$\text{or} \qquad \upsilon = \frac{\lambda}{T} \qquad \text{or} \qquad \boxed{\upsilon = \nu\lambda}$$

So, velocity of wave is the product of frequency and wavelength of the wave. This relation holds for longitudinal as well as transverse waves.

5.10. SOUND

Hearing, like sight, touch, taste etc. is a primary sensation. The term 'sound' is used in two ways. One is the sensation of hearing and another is the physical cause which produces that sensation. When we say that we hear the sound of chirping birds, we refer to this sensation. But when we say that sound travels in air at a speed of 340 m s^{-1}, we refer to the waves of sound which are external to our system of hearing. This is the physical sense in which we use the term 'sound'. The other one is the physiological sense in which we use the term 'sound'.

Sound *may be defined as the physical cause which enables us to have the sensation of hearing.*

Both sound and light are associated with wave motion. Light waves are electromagnetic waves propagating in free space at a tremendous speed of three lakh kilometre per second. On the other hand, sound is a mechanical wave motion, in an elastic medium, moving with a small speed of about 340 m s^{-1} nearly. Further, whereas light does not require any medium to pass through, sound cannot travel in vacuum.

Sound is produced by the vibrations of sounding body. Our ear is not sensitive to all such vibrations. Our range of hearing, *i.e., audible range* is from 20 Hz to 20,000 Hz. Any vibration with a frequency greater than 20,000 Hz is called an *ultrasonic vibration.* A bat produces ultrasonic vibrations which are beyond the range of human hearing. The word 'ultrasonic' should not be confused with supersonic. Any object moving with a speed greater than the speed of sound is said to move with a *supersonic speed.*

Sound requires a material medium for propagation. If there is no material medium between two points as in vacuum, sound cannot travel from one point to another.

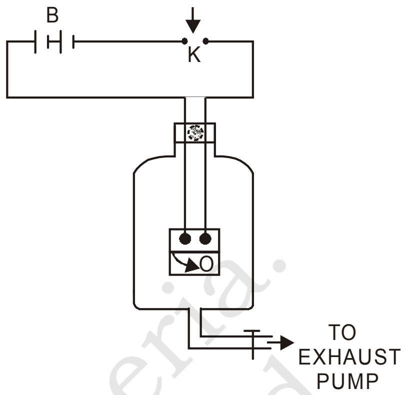


Fig. 5.8. Sound does not travel in vacuum

Example 1. *The audible frequency range of a human ear is 20 Hz – 20 kHz. Convert this into the corresponding wavelength range. Take the speed of sound in air at ordinary temperature to be 340 m s⁻¹.*

Solution. Lower limit of wavelength, $\lambda_{min.} = \dfrac{v}{\nu_{max.}}$

or $$\lambda_{min.} = \frac{340 \text{ m s}^{-1}}{20 \times 10^3 \text{ Hz}} = 17 \times 10^{-3} \text{m} = \mathbf{17 \text{ mm}}$$

Upper limit of wavelength, $$\lambda_{max.} = \frac{v}{\nu_{min.}} = \frac{340 \text{ m s}^{-1}}{20 \text{ s}^{-1}} \text{m} = \mathbf{17 \text{ m}}$$

Example 2. *An observer standing at a sea-coast observes 54 waves reaching the coast per minute. If the wavelength of the waves is 10 m, find the velocity of the waves.*

Solution. $$\nu = \frac{54}{60} \text{s}^{-1}; \ \lambda = 10 \text{ m}$$

$$v = \nu\lambda = \frac{54}{60} \times 10 \text{ m s}^{-1} = \mathbf{9 \text{ m s}^{-1}}$$

5.11. SPEED OF WAVE MOTION

(i) The speed of **transverse wave in a solid** is given by:

$$v = \sqrt{\frac{\eta}{\rho}},$$

where η is the modulus of rigidity of the material and ρ is its density.

(*ii*) The speed of **transverse waves in a stretched string** is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension in the string and μ is the linear mass density, *i.e., mass per unit length* of the string. In SI units, T is measured in newton and 'μ' in kg m^{-1}.

Let diameter of a wire = D ; Density of material of wire = ρ.

Then, μ = mass per unit length of wire

= volume of unit length × density

= cross-sectional area × unit length × density

$$= \pi\left(\frac{D}{2}\right)^2 \times 1 \times \rho$$

$$\therefore \qquad v = \sqrt{\frac{T}{\pi\left(\frac{D^2}{4}\right)\times\rho}} = \frac{2}{D}\sqrt{\frac{T}{\pi\rho}}$$

(*iii*) Speed of longitudinal waves in solids, liquids and gases

Newton, on the basis of theoretical considerations, deduced the following formula for the velocity of longitudinal waves in an elastic medium.

$$v = \sqrt{\frac{E}{\rho}}$$

where E is the elasticity of the medium and ρ is the density of the undisturbed medium. In the case of solids, E represents the Young's modulus of elasticity. In the case of liquids and gases, E represents the bulk modulus of elasticity.

(*iv*) When sound waves propagate through **a long thin rod**, the length of the rod decreases in the region of compression and increases in the region of rarefaction. The only type of strain involved in this is '*longitudinal strain*'. Therefore, the only modulus of elasticity to be considered in this case is 'Young's modulus of elasticity'. The velocity of sound in a long thin rod is given by,

$$v = \sqrt{\frac{Y}{\rho}}$$

Here, ρ is the density of the material of the rod.

(*v*) The velocity of sound in a **liquid** is given by

$v = \sqrt{\frac{B}{\rho}}$ where B is the bulk modulus of elasticity and ρ is the density of the liquid.

Example 3. *Find the speed of transverse waves in a copper wire having a cross-sectional area of 1 mm² under the tension produced by 1 kg wt. The relative density of copper = 8.93.*

Solution.

$$a = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2,$$
$$\rho = 8.93 \times 10^3 \text{ kg m}^{-3}$$
$$T = 1 \text{ kg wt} = 9.8 \text{ N},$$

mass/length,

$$\mu = 10^{-6} \times 1 \times 8.93 \times 10^3 \text{ kg m}^{-1}$$
$$= 8.93 \times 10^{-3} \text{ kg m}^{-1}$$
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{9.8}{8.93 \times 10^{-3}}} \text{ m s}^{-1} = \mathbf{33.13 \text{ m s}^{-1}}$$

Example 4. *Deduce the velocity of longitudinal waves in a metal rod. Given : modulus of elasticity = 7.5 × 10¹⁰ N m⁻² and density = 2.7 × 10³ kg m⁻³.*

Solution.

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7.5 \times 10^{10}}{2.7 \times 10^3}} \text{ m s}^{-1}$$
$$= \mathbf{5.27 \times 10^3 \text{ m s}^{-1}}$$

Example 5. *Determine the speed of sound in a liquid of density 8000 kg m⁻³. Given : bulk modulus = 2 × 10⁹ N m⁻².*

Solution.

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2 \times 10^9}{8000}} \text{ m s}^{-1} = \mathbf{500 \text{ m s}^{-1}}$$

5.12. NEWTON'S FORMULA FOR THE VELOCITY OF SOUND WAVES IN AIR

Newton assumed that sound waves travel in air under isothermal conditions, *i.e.*, temperature remains constant. So, the changes in pressure and volume obey Boyle's law.

$\therefore$ $$PV = \text{constant}$$

Differentiating, $PdV + VdP = 0$ or $PdV = -VdP$

or $$P = -\frac{dP}{dV/V} = \frac{\text{stress}}{\text{strain}} = \text{(isothermal) elasticity } B_i$$

Now, $$v = \sqrt{\frac{B_i}{\rho}} = \sqrt{\frac{P}{\rho}}$$

which is Newton's formula for the velocity of sound waves in air or in a gas.

Let us apply this formula to calculate the velocity of sound in air at NTP.

At NTP, density ρ of air = 1.293 kg m^{-3}

and pressure, $P = 0.76$ m of Hg column

$= 0.76 \times 13600 \times 9.8$ Nm^{-2}

($\because$ $P = hdg$ and $d_{Hg} = 13600$ kg m^{-3})

$\therefore$ $$v = \sqrt{\frac{0.76 \times 13600 \times 9.8}{1.293}} \text{ m s}^{-1} \approx 280 \text{ m s}^{-1}$$

This value is nearly 16% less than the experimental value of 332 m s^{-1}. This discrepancy could not be satisfactorily explained by Newton.

5.13. LAPLACE'S CORRECTION

Laplace, a French mathematician, suggested that *sound waves travel in air under adiabatic conditions and not under isothermal conditions* as suggested by Newton. He gave the following two reasons for this.

(i) When sound waves travel in air, the changes in volume and pressure take place rapidly. ***(ii)*** Air or gas is a bad conductor of heat.

Due to both these factors, the compressed air becomes warm and stays warm whereas the rarefied air suddenly cools and stays cool. For adiabatic changes in pressure and volume,

$$PV^\gamma = \text{constant}$$

On differentiation, $P\gamma V^{\gamma-1}dV + V^\gamma dP = 0$

or $$\gamma P = -\frac{V^\gamma dP}{V^{\gamma-1}dV} = -\frac{VdP}{dV} = -\frac{dP}{\frac{dV}{V}} = B_a,$$

where B_a is adiabatic elasticity.

Now, $$v = \sqrt{\frac{B_a}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

which is Laplace's corrected formula for velocity of sound waves in air or gas.

Again, $v = \sqrt{\gamma} \times \sqrt{\frac{P}{\rho}} = \sqrt{1.41} \times 280 \text{ m s}^{-1} = 332.5 \text{ m s}^{-1}$

This result agrees very well with the experimental value of 332 m s^{-1}. This establishes the correctness of Laplace's formula.

5.14. FACTORS AFFECTING THE VELOCITY OF SOUND IN GASES

(*i*) Effect of change in pressure

At constant temperature, PV = constant (Boyle's law)

or $$\frac{Pm}{\rho} = \text{constant}$$

where m is the mass of the gas and ρ is its density.

or $$\frac{P}{\rho} = \text{constant} \qquad [\because m \text{ is constant.}]$$

or $$\frac{\gamma P}{\rho} = \text{constant} \qquad [\because \gamma \text{ is also constant.}]$$

$\therefore$ $v \left(= \sqrt{\frac{\gamma P}{\rho}} \right)$ is also constant.

So, *if the temperature remains constant, the change in pressure has no effect on the velocity of sound in a gas.*

Clearly, the velocity of sound in a gas is independent of pressure, provided temperature remains constant.

(*ii*) Effect of change in temperature

Let v_0 and v_t be the velocity of sound in a gas 0°C and t°C respectively. Let γ and P remain the same at both temperatures.

Thus, $$v_0 = \sqrt{\frac{\gamma P}{\rho_0}} \quad \text{and} \quad v_t = \sqrt{\frac{\gamma P}{\rho_t}}$$

Dividing, $\dfrac{v_t}{v_0} = \sqrt{\dfrac{\gamma P}{\rho_t}} \times \sqrt{\dfrac{\rho_0}{\gamma P}} = \sqrt{\dfrac{\rho_0}{\rho_t}}$ ...(1)

Let V_0 and ρ_0 be the volume and density respectively of a given mass m of gas at 0°C. Let V_t and ρ_t be the volume and density respectively for the same mass m of gas at t°C.

Then, $V_t\rho_t = V_0\rho_0 = m$ or $\dfrac{V_t}{V_0} = \dfrac{\rho_0}{\rho_t}$

But $\dfrac{V_t}{V_0} = \dfrac{T}{T_0}$ (Charle's law)

where T_0 and T are the absolute temperatures corresponding to 0°C and t°C respectively.

$\therefore \dfrac{T}{T_0} = \dfrac{\rho_0}{\rho_t}$ $\therefore$ $\dfrac{v_t}{v_0} = \sqrt{\dfrac{T}{T_0}}$ ...(2) [from equation (1)]

So, the velocity of sound varies directly as the square root of the absolute temperature of the gas. This explains as to why sound travels faster on a hot summer day than on a cold winter day.

Temperature coefficient of velocity of sound

From equation (2), $\dfrac{v_t}{v_0} = \sqrt{\dfrac{273+t}{273+0}} = \sqrt{\dfrac{273+t}{273}}$

or $$\frac{v_t}{v_0} = \sqrt{1+\frac{t}{273}} = \left(1+\frac{t}{273}\right)^{1/2}$$

Assume t to be small. Expanding the right hand side of the above equation by Binomial theorem and neglecting squares and higher powers of $\dfrac{t}{273}$, we get

$$\frac{v_t}{v_0} = 1 + \frac{1}{2} \times \frac{t}{273} = 1 + \frac{t}{546}$$

or $$v_t = v_0\left(1+\frac{t}{546}\right) = v_0 + \frac{v_0}{546}t$$

or $$v_t - v_0 = \frac{v_0 t}{546} = 332 \times \frac{t}{546}\text{ m s}^{-1} \qquad [v_0 = 332\text{ m s}^{-1}]$$

or $$v_t - v_0 = 0.608 \times t\,\text{m s}^{-1} = 0.61 \times t\,\text{m s}^{-1}$$

Temperature coefficient of velocity of sound,

$$\alpha = \frac{v_t - v_0}{t} = 0.61 \text{ m s}^{-1}\,{}^\circ\text{C}^{-1}$$

When $t = 1°\text{C}$, then $v_t - v_0 = 0.61 \text{ m s}^{-1}$ or 61 cm s^{-1}

So, the velocity of sound increases by 0.61 m s^{-1} for every one degree centigrade rise of temperature. This is known as the temperature coefficient of velocity of sound in air.

(*iii*) Effect of change in density

Consider two different gases at the same temperature and pressure with different densities.

Then, $v_1 = \sqrt{\frac{\gamma_1 P}{\rho_1}}$ and $v_2 = \sqrt{\frac{\gamma_2 P}{\rho_2}}$ or $\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1}{\gamma_2} \times \frac{\rho_2}{\rho_1}}$

For diatomic gases, $\gamma_1 = \gamma_2$. $\therefore$ $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

So, the velocity of sound in a gas is inversely proportional to the square root of the density of the gas.

Illustration. The density of oxygen is 16 times the density of hydrogen.

$$\therefore \quad \frac{v_H}{v_{O_2}} = \sqrt{\frac{\rho_{O_2}}{\rho_H}} = \sqrt{\frac{16\rho_H}{\rho_H}} = 4$$

Thus, all other things being equal, sound travels four times faster in hydrogen than in oxygen.

(*iv*) Effect of humidity

We know that humid air contains a large proportion of water vapour. So, the density ρ_m of moist air is less than the density ρ_d of dry air.

$$\frac{\rho_d}{\rho_m} = 1.6 \qquad \text{Also, } \frac{\gamma_m}{\gamma_d} = 0.9$$

Let v_m and v_d be the velocities of sound in moist air and dry air respectively.

Then, $$v_m = \sqrt{\frac{\gamma_m P}{\rho_m}} \quad \text{and} \quad v_d = \sqrt{\frac{\gamma_d P}{\rho_d}}$$

$$\frac{v_m}{v_d} = \sqrt{\frac{\gamma_m P}{\rho_m}} \times \sqrt{\frac{\rho_d}{\gamma_d P}} = \sqrt{\frac{\gamma_m}{\gamma_d} \times \frac{\rho_d}{\rho_m}}$$

or $$\frac{v_m}{v_d} > 1 \quad \text{or} \quad v_m > v_d$$

So, sound travels faster in moist air than in dry air. This explains as to why sound travels faster on a rainy day than on a dry day.

(v) Effect of wind

Let wind travel with a velocity w making an angle θ with the direction of propagation of sound [Fig. 5.9]. Then, the effective velocity of sound will be ($v + w \cos \theta$).

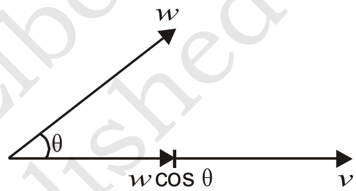


Fig. 5.9. Effect of wind on velocity of sound

If the wind blows in the direction of sound, then the velocity of sound will be increased from v to ($v + w$). If the wind blows in a direction opposite to the direction of propagation of sound, then the velocity of sound is decreased from v to ($v - w$). If wind blows perpendicular to the direction of sound, then $\theta = 90°$ and $\cos \theta = \cos 90° = 0$. So, there will be no effect on velocity of sound.

Example 6. *At what temperature will the velocity of sound in hydrogen be twice as much as that at 27°C ?*

Solution. $$\frac{v_t}{v_{27}} = \sqrt{\frac{273+t}{273+27}}$$

or $$\frac{2 \times v_{27}}{v_{27}} = \sqrt{\frac{273+t}{300}} \quad \text{or} \quad 4 = \frac{273+t}{300}$$

or $$273 + t = 1200 \quad \text{or} \quad t = \mathbf{927°C}$$

Example 7. *At normal temperature and pressure, the speed of sound in air is 332 m s^{-1}. What will be the speed of sound in hydrogen (i) at normal temperature and pressure, (ii) at 819°C temperature and 4 atmospheric pressure ? Given : air is 16 times heavier than hydrogen.*

Solution. (i) Let v_a and v_h represent the speeds of sound in air and hydrogen respectively.

$$v_a = \sqrt{\frac{\gamma P}{d_a}} \text{ and } v_h = \sqrt{\frac{\gamma P}{d_h}}$$

Now, $$\frac{v_a}{v_h} = \sqrt{\frac{d_h}{d_a}} \text{ But } \frac{d_h}{d_a} = \frac{1}{16}$$

$\therefore$ $$\frac{v_a}{v_h} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

or $$v_h = 4v_a = 4 \times 332 \text{ m s}^{-1} = \mathbf{1328 \text{ m s}^{-1}}$$

(*ii*) Pressure has no effect on the velocity of sound.

$$\frac{v_{819}}{v_0} = \sqrt{\frac{273+819}{273+0}} = \sqrt{\frac{1092}{273}} = \sqrt{4} = 2$$

or $$v_{819} = 2 \times v_0 = 2 \times 1328 \text{ m s}^{-1} = \mathbf{2656 \text{ m s}^{-1}}$$

5.15. PRINCIPLE OF SUPERPOSITION OF WAVES

Statement. ***The displacement due to a number of waves acting simultaneously at a point in a medium is the sum of the displacement vectors due to each one of them acting separately.***

Since displacements are either positive or negative, therefore, the net displacement is an algebraic sum of the individual displacements.

An interesting property of a wave is that it preserves its individuality when travelling through space. Each wave behaves as if it has nothing to do with other waves. This fact is amply illustrated by the following examples.

(*i*) In an orchestra, different musical instruments are playing simultaneously. But we can detect the note produced by an individual instrument.

(*ii*) Different radio waves cross the antenna. But we can pick up any given frequency.

These examples establish the independent behaviour of a wave. *Huygen's principle of superposition* is a natural consequence of the independent behaviour of a wave.

Consider two pulses (in a string) approaching each other as shown in Fig. 5.10 (*a*). When the pulses cross each other, they combine to produce a zero resultant throughout the string as shown in Fig. 5.10 (*b*). After crossing each other, they again begin to travel independently as if nothing had happened as shown in Fig. 5.10 (*c*).

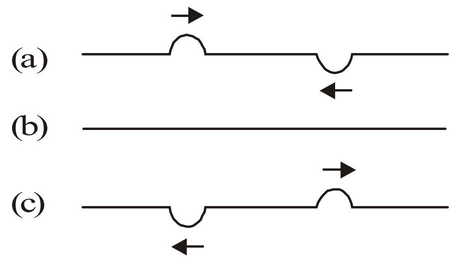


Fig. 5.10. Two pulses having equal and opposite displacements moving in opposite directions. The overlapping pulses add up to zero displacement in (*b*).

Following are the *three consequences* of the principle of superposition of waves.

(*i*) Two waves of the same frequency move with the same velocity in the same direction. This gives rise to the phenomenon of **interference of waves.**

(*ii*) Two waves of identical frequencies and amplitudes travel along the same path with the same speeds in the opposite directions. This gives rise to **stationary waves.**

(*iii*) Two waves of slightly different frequencies moving with the same velocity in the same direction give rise to the phenomenon of **beats.**

5.16. DISPLACEMENT RELATION FOR A SIMPLE HARMONIC PLANE PROGRESSIVE WAVE OR EQUATION OF PROGRESSIVE WAVE

*A **progressive wave** is one which travels in a given direction with constant amplitude, i.e., without attenuation.*

In the following treatment, we shall consider transverse wave motion. However, the treatment is valid for longitudinal wave motion also.

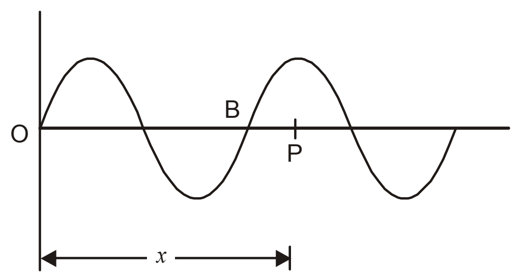


Fig. 5.11. Plane progressive wave

Let a plane wave originate at O as shown in Fig. 5.11. Let it proceed from left to right in an elastic medium. As discussed earlier, particles of the medium shall execute SHM of the same amplitude and time period about its mean position. Let us count time from the instant the particle at O crosses its mean position in the positive direction of Y-axis. The displacement y of the particle at any time t is given by

$$y(0, t) = \mathrm{A} \sin \omega t$$

where A and ω represent the amplitude and angular frequency respectively of simple harmonic motion executed by the particle at O.

Since the disturbance is handed over from one particle to the next therefore there is a gradual fall in phase from left to right, *i.e.,* in the direction of motion. Let the phase of particle at P *lag behind* the phase of particle at O by ϕ. Then, the displacement of particle at P at any time t is given by

$$y(x, t) = \mathrm{A} \sin (\omega t - \phi) \quad \ldots(1)$$

At B, which is one wavelength λ apart from O, the phase difference is 2π. In other words, particles at O and B have the same phase of vibration.

At a distance λ, the phase changes by 2π.

At a distance x, the phase changes by $\frac{2\pi}{\lambda}x$.

$$\therefore \qquad \phi = \frac{2\pi}{\lambda}x$$

where x is the distance of P from O.

From equation (1), $y(x, t) = \mathrm{A} \sin\left(\omega t - \frac{2\pi}{\lambda}x\right) \quad \ldots(2)$

Now, $\omega = \frac{2\pi}{\mathrm{T}}$ and $\mathrm{T} = \frac{\lambda}{\upsilon}$

$$\therefore \qquad \omega = \frac{2\pi}{\lambda}\upsilon$$

where υ is called the wave velocity or phase velocity.

From equation (2), $y(x, t) = \mathrm{A} \sin\left(\frac{2\pi}{\lambda}\upsilon t - \frac{2\pi}{\lambda}x\right)$

or $$y(x, t) = \mathrm{A} \sin \frac{2\pi}{\lambda}(\upsilon t - x) \quad \ldots(3)$$

Also, $$y(x, t) = \mathrm{A} \sin 2\pi\left(\frac{\upsilon}{\lambda}t - \frac{x}{\lambda}\right)$$

or $$y(x, t) = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad ...(4)$$

Again, from equation (2), $y(x, t) = A\sin(\omega t - kx)$ ...(5) $\left[\because \frac{2\pi}{\lambda} = k\right]$

Discussion. (*i*) While arriving at the wave equation, we have made a particular choice of t = 0. The origin of time has been chosen at an instant when the left end x = 0 is crossing its mean position y = 0 and is moving up. For a **general choice of the origin of time,** we need to add a phase constant (also known as initial phase angle) ϕ_0 so that equation (5) will be,

$$y = A \sin [(\omega t - kx) + \phi_0] \quad ...(6)$$

For $\phi_0 = \frac{\pi}{2}$, $y = A \sin\left[(\omega t - kx) + \frac{\pi}{2}\right]$

or $$y = A \cos (\omega t - kx) \quad ...(7)$$

Using $\cos(-\theta) = \cos\theta$,

$$y = A \cos (kx - \omega t) \quad ...(8)$$

For $\phi_0 = \pi$, $y = A \sin [(\omega t - kx) + \pi]$

or $y = -A \sin (\omega t - kx)$

Using $\sin(-\theta) = -\sin\theta$,

$$y = A \sin (kx - \omega t) \quad ...(9)$$

For $\phi_0 = \frac{3\pi}{2}$, $y = A \sin\left[(\omega t - kx) + \frac{3\pi}{2}\right]$

$$y = -A \cos (\omega t - kx)$$

For $\phi_0 = 2\pi$, $y = A \sin [(\omega t - kx) + 2\pi]$

or $$y = A \sin (\omega t - kx) \quad ...(10)$$

5.17. AMPLITUDE OF WAVE

The amplitude of a wave is the magnitude of maximum displacement of the constituents of the medium from their equilibrium positions as the wave passes through them.

In the equation of the travelling wave, $y(x, t)$ varies between A and – A. This is because the sine function varies between 1 and – 1. Without any loss of generality, we can take A to be a positive constant. Then A represents the maximum displacement of the constituents of the medium from their equilibrium position. Note that the displacement y may be positive or negative, but A is positive. It is called the **amplitude of the wave.**

5.18. PHASE OF WAVE

The phase of a wave is a quantity which determines the displacement of the wave at any position and at any instant. Mathematically, the quantity appearing as the argument of the sine function in the equation of the travelling wave is called the **phase of the wave**. It is denoted by ϕ.

Considering equation $y(x, t) = A \sin(\omega t - kx + \phi_0)$,

$$\phi = \omega t - kx + \phi_0$$

Clearly, ϕ_0 is the phase at $x = 0$ and $t = 0$. Hence ϕ_0 is called the initial phase angle. By suitable choice of origin on the x-axis and the initial time, it is possible to have $\phi_0 = 0$. Thus, there is no loss of generality in dropping ϕ_0 *i.e.,* in considering equations of travelling wave with $\phi_0 = 0$.

5.19. WAVELENGTH OF WAVE AND ANGULAR WAVE NUMBER

The minimum distance between two points having the same phase is called the wavelength of the wave. It is usually denoted by λ.

For simplicity, we can choose points of the same phase to be crests or troughs. The wavelength is then the distance between two consecutive crests or troughs in a wave. Considering the equation $y(x, t) = A \sin(kx - \omega t)$, the displacement at $t = 0$ is given by

$$y(x, 0) = a \sin kx$$

Since the sine function repeats its value after every 2π change in angle,

$$\therefore \qquad \sin kx = \sin(kx + 2n\pi) = \sin k\left(x + \frac{2n\pi}{k}\right)$$

That is the displacements at points x and at $x + \dfrac{2n\pi}{k}$ are the same, where n = 1, 2, 3, ... The least distance between points with the same displacement (at any given instant of time) is obtained by taking n = 1. λ is then given by

$$\lambda = \frac{2\pi}{k} \quad \text{or} \quad k = \frac{2\pi}{\lambda}$$

k is the angular wave number or propagation constant. Its SI unit is radian per metre or rad m^{-1}. Sometimes, k is simply measured in m^{-1}. *Angular wave number is 2π times the number of waves that can be accomodated per unit length.*

5.20. PERIOD, ANGULAR FREQUENCY AND FREQUENCY

Time period of a wave is equal to the time taken by the wave to travel a distance equal to one wavelength. It is denoted by T.

Frequency of a wave is the number of complete wavelengths traversed by the wave in one second. It is denoted by ν.

Angular frequency of a wave is 2π times the frequency of the wave.

Fig. 5.12 shows the sinusoidal plot of a travelling wave. It helps us to describe the displacement of an element (at any fixed location) of the medium as a function of time. Let us consider the equation : $y(x, t)$ = A cos $(kx - \omega t)$ and monitor the motion of the element, say at $x = 0$.

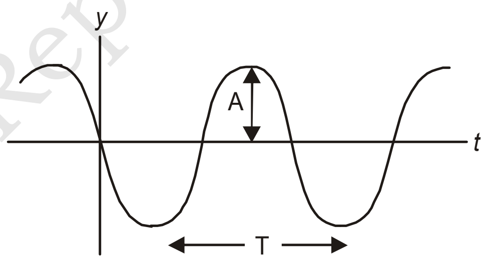


Fig. 5.12. An element of a string at a fixed location oscillates in time with amplitude A and period T, as the wave passes over it

$$y(0, t) = \text{A} \sin(-\omega t)$$
$$= -\text{A} \sin \omega t$$

Now, the period of oscillation of the wave is the time it takes for an element to complete one full oscillation. That is

$$-\text{A} \sin \omega t = -\text{A} \sin \omega(t + \text{T})$$
$$= -\text{A} \sin(\omega t + \omega \text{T})$$

Since sine function repeats after every 2π.

$$\therefore \quad \omega\text{T} = 2\pi \quad \text{or} \quad \omega = \frac{2\pi}{\text{T}}$$

ω is called the angular frequency of the wave. Its SI units is rad s^{-1}. The frequency ν is the number of oscillations per second. Therefore,

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

ν is usually measured in hertz.

Example 8. *A wave travelling along a string is described by,*

$y(x, t) = 0.005 \sin (80.0\ x - 3.0\ t)$,

in which the numerical constants are in SI units (0.005 m, 80.0 rad m^{-1}, and 3.0 rad s^{-1}). Calculate (a) the amplitude, (b) the wavelength, and (c) the period and frequency of the wave. Also, calculate the displacement y of the wave at a distance x = 30.0 cm and time t = 20 s?

Solution. On comparing the given displacement equation with

$$y(x, t) = y_m \sin (kx - \omega t),$$

we find

(a) the amplitude of the wave is 0.005 m = **5 mm**

(b) the angular wave number k and angular frequency ω are

$$k = 80.0 \text{ rad m}^{-1} \text{ and } \omega = 3.0 \text{ rad s}^{-1}$$

We then relate the wavelength λ to k through $\lambda = 2\pi / k$

$$= \frac{2\pi \text{ rad}}{80.0 \text{ rad m}^{-1}} = \mathbf{7.85\ cm}$$

(c) Now we relate T to ω by the relation T = $2\pi/\omega$

$$= \frac{2\pi \text{ rad}}{3.0 \text{ rad s}^{-1}} = \mathbf{2.09\ s}$$

and frequency, ν = 1/T = **0.48 Hz**

The displacement y at x = 30.0 cm and time t = 20 s is given by

$$y = 0.005 \text{ m} \sin (80.0 \times 0.3 - 3.0 \times 20)$$

$$= 0.005 \text{ m} \sin (-36 \text{ rad}) = \mathbf{5\ mm}$$

Example 9. *Given :* $y = 0.8 \sin 16\pi \left[t + \frac{x}{40}\right]$ *metre. Calculate the wavelength and the velocity of the wave represented by this equation.*

Solution. Rewriting the given equation,

$$y = 0.8 \sin 2\pi \left[8t + \frac{8x}{40}\right] \quad \text{or} \quad y = 0.8 \sin 2\pi \left[8t + \frac{x}{5}\right]$$

Comparing with $y = A \sin 2\pi\left[\frac{t}{T} + \frac{x}{\lambda}\right]$, we get

$\frac{1}{T} = 8$ or ν = 8 Hz, λ = **5 m**

Velocity, $\upsilon = \nu\lambda$ = **40 m s^{-1}**

5.21. FUNDAMENTAL MODE AND HARMONICS OF A STRING

(*i*) First or Fundamental mode of vibration. In this mode of vibration, the string vibrates as a whole in one segment (Fig. 5.13*a*). There are two nodes and one antinode. If λ_1 is the wavelength of the standing wave, then $\frac{\lambda_1}{2}$ = L or λ_1 = 2L. The corresponding frequency of vibration is given by

$$\nu_1 = \frac{\upsilon}{\lambda_1} = \frac{\upsilon}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

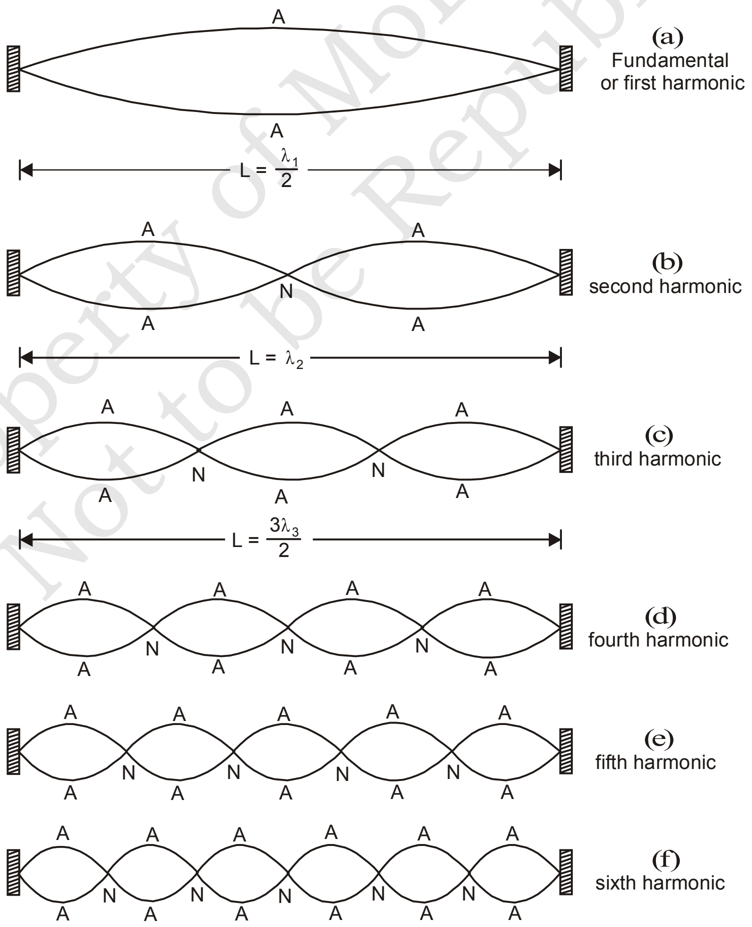


Fig. 5.13. Stationary waves in a stretched string fixed at both ends.

This is the lowest possible natural frequency of the string. This frequency is called fundamental frequency. The sound or note produced is called ***fundamental note*** or ***fundamental tone*** or ***first harmonic.***

(*ii*) Second mode of vibration. In this mode of vibration, the string vibrates in two segments or loops of equal length (Fig. 5.13*b*). There are three nodes and two antinodes. If λ_2 is the wavelength of the standing wave, then λ_2 = L. The corresponding frequency of vibration is given by

$$\nu_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right)$$

$$= 2\nu_1 = 2\left[\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right]$$

The frequency of vibration of the string becomes twice the fundamental frequency. The note produced is called **first overtone** or **second harmonic**.

(*iii*) Third mode of vibration. In this mode of vibration, the string vibrates in three segments or loops of equal length (Fig. 5.13*c*). If λ_3 is the wavelength, then $L = \frac{3\lambda_3}{2}$ or $\lambda_3 = \frac{2L}{3}$. The corresponding frequency is

$$\nu_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3\left(\frac{v}{2L}\right) = 3\nu_1 = 3\left[\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right]$$

The frequency of vibration of the string becomes three times the natural frequency. The note produced is called **second overtone** or **third harmonic**.

Figs. 5.13(*d*), (*e*) and (*f*) show fourth, fifth and sixth mode of vibration.

In general, if the string is made to vibrate in n loops or segments, then $L = n\frac{\lambda_n}{2}$ or $\lambda_n = \frac{2L}{n}$. $\nu_n = \frac{v}{\lambda_n} = n\frac{v}{2L}$ or $\nu_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

Positions of Nodes. In the first mode, there are two nodes. These are located at x = 0, L. In the second mode, there are three nodes. These are located at $x = 0, \frac{L}{2}$, L. In the third mode, there are four nodes located at $x = 0, \frac{L}{3}, \frac{2L}{3}$, L. In the nth mode, there will be $(n + 1)$ nodes located at $x = 0, \frac{L}{n}, \frac{2L}{n}, \frac{3L}{n}, \ldots\ldots, L$.

Positions of Antinodes. In the first mode, there is one antinode located at $x = \frac{L}{2}$. In the second mode, there are two antinodes located at $x = \frac{L}{4}, \frac{3L}{4}$. In the third mode, there are three antinodes located at $x = \frac{L}{6}, \frac{3L}{4}, \frac{5L}{6}$. In the *n*th mode, there are *n* antinodes located at $x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \ldots, \frac{(2n-1)L}{2n}$.

5.22. LAWS OF VIBRATIONS OF STRINGS

We know that $$\nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

The following laws of vibrations of strings follow from this equation.

(*i*) Law of length. If the tension in a given string remains constant, then the fundamental frequency varies inversely as the length.

$$\nu \propto \frac{1}{L}$$

If the length of the string is halved, the frequency is doubled.

(*ii*) Law of tension. For a string of given length and material, the fundamental frequency varies directly as the square root of the tension.

$$\nu \propto \sqrt{T}$$

If the tension is increased four times, the frequency of the note becomes double.

(*iii*) Law of mass. For a string of given length and fixed tension, the frequency varies inversely as the square root of linear density (mass per unit length) of the string.

$$\therefore \qquad \nu \propto \frac{1}{\sqrt{\mu}}$$

If linear density is quadrupled, the frequency is halved.

Consider a string of diameter D. Let ρ be the density of material of the string.

Cross-sectional area of the string = $\frac{\pi D^2}{4}$

Volume of unit length of string = $\frac{\pi D^2}{4} \times 1 = \frac{\pi D^2}{4}$

Mass per unit length = Volume of unit length × density

$$\therefore \qquad \mu = \frac{\pi D^2}{4} \times \rho$$

$$\therefore \qquad \nu = \frac{1}{2L}\sqrt{\frac{T \times 4}{\pi D^2 \rho}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

This leads to following two laws. Of course, both these laws are contained in the *law of mass* stated earlier.

1. Law of diameter. For a string of given length and tension, the frequency is inversely proportional to the diameter of the string.

$$\nu \propto \frac{1}{D}$$

So, thinner the string, higher is the frequency of vibration.

2. Law of density. For a string of given length, diameter and tension, the frequency is inversely proportional to the square root of the density of the material of the string.

$$\nu \propto \frac{1}{\sqrt{\rho}}$$

Smaller the density, higher is the frequency of vibration.

Example 10. *A steel wire 0.72 m long has a mass of 5.0 × 10^{-3} kg. If the wire is under a tension of 60 N, what is the speed of transverse waves in the wire ?*

Solution. Mass per unit length of wire,

$$\mu = \frac{5.0 \times 10^{-3}\ \text{kg}}{0.72\ \text{m}} = 6.9 \times 10^{-3}\ \text{kg m}^{-1}$$

Tension, $T = 60$ N

Speed of wave on the wire, $v = \sqrt{\frac{T}{\mu}}$

$$= \sqrt{\frac{60}{6.9 \times 10^{-3}}}\ \text{m s}^{-1} = \mathbf{93.25\ m\ s^{-1}}$$

Example 11. *A 100 cm long wire of mass 40 g supports a mass of 1.6 kg as shown in Fig. 5.14. Find the fundamental frequency of the portion of the string between the wall and the pulley. Take g = 10 m s*$^{-2}$.

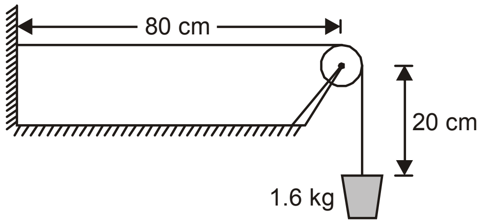


Fig. 5.14

Solution. T = 1.6 kg wt = 1.6 × 10 = 16 N

$$\mu = \frac{40 \times 10^{-3}}{1} = 0.04 \text{ kg m}^{-1}$$

$$L = (100 - 20) \text{ cm} = 0.8 \text{ m}$$

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 0.8}\sqrt{\frac{16}{0.04}} \text{ Hz} = \mathbf{12.5 \text{ Hz}}$$

Example 12. *A sonometer wire carries a brass weight (specific gravity = 8) at its end and has a fundamental frequency of 320 Hz. What would be its frequency if this weight is completely immersed in water?*

Solution. When the weight is immersed in water, buoyancy is $\frac{T}{8}$, where T is the tension in the wire.

$$\text{Net tension} = T - \frac{T}{8} = \frac{7T}{8}$$

$$\nu = 320\sqrt{\frac{7}{8}} \text{ Hz} = \mathbf{293.3 \text{ Hz}}$$

5.23. VIBRATIONS OF AIR COLUMN

Open Organ Pipe

(*i*) Introduction. It is a wind instrument in which sound is produced by setting into vibrations an air column in it.

(*ii*) Construction. It consists of a wooden or metallic hollow tube called resonator (R). A narrow tapering opening called mouth-piece (m) is provided at one end of the resonator as shown in Fig. 5.15. A slanting solid called bevel (B) is fitted near the mouth-piece. The height of the bevel is

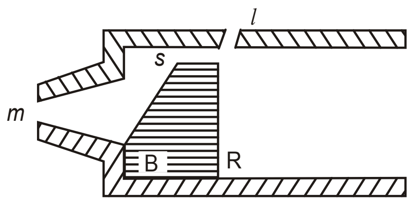


Fig. 5.15. Open organ pipe

such that there is only a narrow slit *s* between the bevel and the wall of the resonator. A sharp edge (l) is provided in the wall of the resonator. This is called the lip of the pipe.

(*iii*) Working. Air is blown into the pipe through the mouth-piece. After striking against the bevel, the air passes through the narrow slit *s* in the form of a thin sheet. This fast moving sheet of air strikes against the lip setting it into vibrations. The vibrating lip produces a sound called edge tone. The frequency of the edge tone depends not only on the pressure with which air is blown into the pipe but also on the distance of the lip from the slit.

(*iv*) Formation of longitudinal stationary waves. When the waves reach the open end of the pipe, they are reflected. This is because the air outside the resonator is rarer than the air inside it. The reflected and the incident waves superpose to give **longitudinal stationary waves** with fixed nodes and antinodes. When the frequency of the vibrating air column in the resonator becomes equal to the frequency of the edge tone, resonance occurs and hence loud sound is produced.

Fundamental Mode and Harmonics of Open Organ Pipe

Since both the ends of the pipe are open therefore the waves are reflected from these ends. However, the particles continue to move in the same direction even after the reflection of the waves at the open ends. So, the particles have maximum displacements at the open ends. Thus, antinodes are formed at the open ends.

Fundamental or First normal mode of vibration

This is the simplest mode of vibration in which the antinodes at the ends are separated by a node in the middle.

In this mode of vibration,

$$\frac{\lambda_1}{2} = L \quad \text{or} \quad \lambda_1 = 2L$$

Frequency, $$\nu_1 = \frac{v}{\lambda_1} \quad \text{or} \quad \nu_1 = \frac{v}{2L}$$

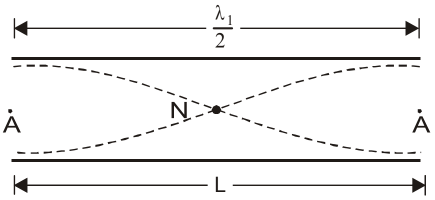


Fig. 5.16. First mode of vibration

Since this is the simplest mode of vibration therefore the sound produced is called **fundamental tone** or **first harmonic.** Longer the resonator, lesser will be the frequency of sound produced.

Second normal mode of vibration

In this mode of vibration, the antinodes at the open ends are separated by two nodes and one antinode [Fig. 5.17].

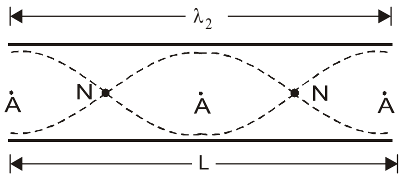


Fig. 5.17. Second mode of vibration

If L be the length of the resonator, then

$$\lambda_2 = L$$

Frequency, $$\nu_2 = \frac{v}{\lambda_2}$$

or $$\nu_2 = 2 \times \frac{v}{2L} \quad \text{or} \quad \nu_2 = 2\nu_1$$

The sound produced in this mode of vibration is called **first overtone** or **second harmonic**. The frequency of first overtone is two times the frequency of the fundamental tone.

Third normal mode of vibration

In this mode of vibration, the antinodes at the open ends are separated by three nodes and two antinodes.

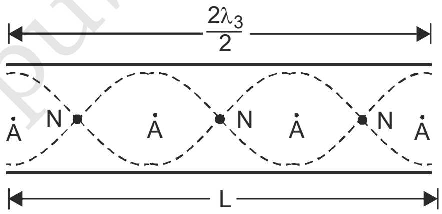


Fig. 5.18. Third mode of vibration

In this mode of vibration,

$$\frac{3\lambda_3}{2} = L \quad \text{or} \quad \lambda_3 = \frac{2L}{3}$$

Frequency, $$\nu_3 = \frac{v}{\lambda_3} = \frac{v}{2L/3}$$

or $$\nu_3 = 3 \times \frac{v}{2L} \quad \text{or} \quad \nu_3 = 3\nu_1$$

The sound produced in this mode of vibration is called **second overtone** or **third harmonic**. Its frequency is three times the fundamental frequency.

By adjusting the pressure with which air is blown into the pipe, the tones of frequencies ν_1, $2\nu_1$, $3\nu_1$, $4\nu_1$, ... can be produced. Thus, the frequencies of different overtones are simple integral multiples of the frequency of fundamental tone.

In general, the frequency of vibration in *n*th normal mode of vibration in an open organ pipe is given by:

$$\boxed{\nu_n = n\nu_1}$$

The note produced in this case is called nth harmonic or $(n - 1)$th overtone. It would contain n nodes and $(n + 1)$ antinodes.

Closed Organ Pipe

Construction. Its construction is similar to that of open organ pipe except that its one end is closed. The waves are reflected from the closed end as the closed end behaves like a denser medium. The incident and the reflected waves superpose to form longitudinal stationary waves having fixed nodes and antinodes. When the frequency of the edge tone is equal to the frequency of vibration of the air column, then the resonance takes place. Consequently, a loud sound is heard.

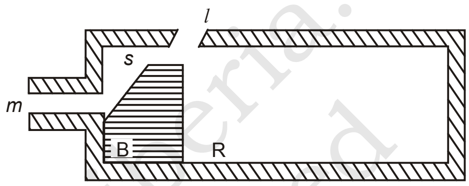


Fig. 5.19. Closed organ pipe

Fundamental Mode and Harmonics of Closed Organ Pipe

When the wave is reflected from the closed end, the direction of motion of the particles changes. So, the displacement is zero at the closed end. Thus, a node is formed at the closed end. On the other hand, an antinode is formed at the open end. This is because the displacement of particles is maximum at the open end.

Fundamental or First normal mode of vibration

This is the simplest mode of vibration in which there is a node at the closed end and an antinode at the open end [Fig. 5.20].

If L be the length of the resonator, then

$$L = \frac{\lambda_1}{4} \quad \text{or} \quad \lambda_1 = 4L$$

Frequency, $$\nu_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

Since this is the simplest mode of vibration therefore the sound produced is called **fundamental tone** or **first harmonic**. Longer the resonator, lesser will be the frequency of sound produced.

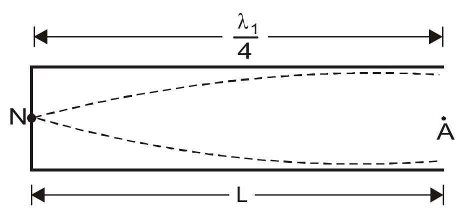


Fig. 5.20. First mode of vibration

Second normal mode of vibration

In this mode of vibration, there is one antinode and one node between a node at the closed end and an antinode at the open end [Fig. 5.21].

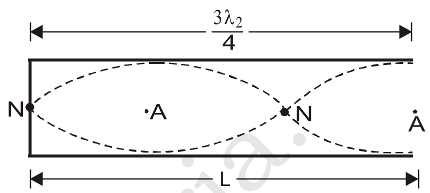


Fig. 5.21. Second mode of vibration

In this case, $\frac{3\lambda_2}{4} = L$ or $\lambda_2 = \frac{4L}{3}$

Frequency, $\nu_2 = \frac{v}{\lambda_2} = \frac{v}{4L/3}$

or $\boxed{\nu_2 = 3 \times \frac{v}{4L}}$ or $\boxed{\nu_2 = 3\nu_1}$

The sound produced in this mode of vibration is called **first overtone** or **third harmonic**. The frequency of the first overtone is three times the frequency of the fundamental tone.

Third normal mode of vibration

In this mode of vibration, there are two nodes and two antinodes between a node at the closed end and an antinode at the open end [Fig. 5.22].

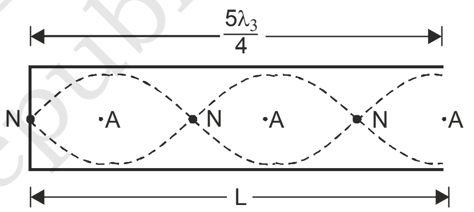


Fig. 5.22. Third mode of vibration

In this case, $L = \frac{5\lambda_3}{4}$ or $\lambda_3 = \frac{4L}{5}$

Frequency, $\nu_3 = \frac{v}{\lambda_3} = \frac{v}{4L/5}$

or $\nu_3 = 5 \times \frac{v}{4L}$ or $\nu_3 = 5\nu_1$

The sound produced in this case is called **second overtone** or **fifth harmonic**. The frequency of the second overtone is five times the frequency of the fundamental tone.

By adjusting the pressure with which air is blown into the pipe, the tones of frequencies ν_1, $3\nu_1$, $5\nu_1$,...... can be produced. Thus, the frequencies of different overtones are odd multiples of the frequency of fundamental tone.

In general, the frequency of vibration in *n*th normal mode of vibration in a closed organ pipe is given by,

$$\nu_n = (2n - 1)\frac{v}{4L} = (2n - 1)\nu_1$$

The note produced in this case is called $(2n - 1)$th harmonic or $(n - 1)$th overtone.

Comparison of closed and open organ pipes

(*i*) Fundamental note in closed pipe has half the frequency of the fundamental note in open pipe.

(*ii*) In a closed pipe, only odd harmonics are present. In an open pipe, all harmonics are present.

(*iii*) The musical sound produced by an open pipe is richer than the musical sound produced by a closed organ pipe.

Example 13. *A closed organ pipe can vibrate at a minimum frequency of 500 Hz. Find the length of the tube. Speed of sound in air = 340 m s⁻¹.*

Solution. $$\nu = \frac{v}{4L}$$

or $$L = \frac{v}{4\nu} = \frac{340}{4 \times 500}\text{ m} = 0.17\text{ m} = \mathbf{17\ cm}$$

Example 14. *An open organ pipe emits a note of frequency 256 Hz which is its fundamental. What would be the smallest frequency produced by a closed pipe of the same length?*

Solution. For open organ pipe, $\nu = \frac{v}{2L}$

$$256 = \frac{v}{2L} \quad \text{or} \quad v = 512\text{ L}$$

For closed organ pipe, $\nu = \frac{v}{4L} = \frac{512\text{ L}}{4L} = \mathbf{128\ Hz}$

5.24. BEATS

When two sounding bodies of nearly the same frequency and same amplitude are sounded together, the resultant sound comprises of alternate maxima and minima.

The phenomenon of alternate waxing and waning of sound at regular intervals is called ***beats.***

The number of beats heard per second is called *beat frequency.* It is equal to the difference in the frequencies of sounding bodies. Beats are heard only when the difference in frequencies of two sounding bodies is not more than ten. This is due to *persistence of hearing.*

The time from each loud sound to the next loud sound is called ***one beat-period.***

Suppose at any place, two sound waves are in the same phase. The amplitudes of the two sound waves will be added up resulting in maximum amplitude. Since intensity is directly proportional to square of amplitude therefore loud sound will be heard.

But since the frequencies are different, even though slightly, one sound wave will start getting out of phase from the other as time passes on. Eventually, the two waves will get *out of phase* with each other. This will produce minimum amplitude resulting in a *faint sound, i.e.,* sound of low intensity. As time further elapses, the phase again goes on changing and again, we get a loud sound. In this way we continue to hear loud and faint sounds alternately. *One loud sound plus one faint sound constitute a beat.*

Analytical treatment of beats. Consider two harmonic sound waves of nearly equal frequencies ν_1 and ν_2. The periodic dips in sound, called beats, will occur with a frequency equal to $(\nu_1 - \nu_2)$.

Let a be the amplitude of each wave. Let us count time from the instant when the two sound waves are in the same phase. The displacements s_1 and s_2 at a point due to the two waves are given by

$$s_1 = a \cos 2\pi\nu_1 t \quad \text{and} \quad s_2 = a \cos 2\pi\nu_2 t$$

For the sake of simplicity, it is assumed here that there is no initial phase difference between the two wave trains. It is further assumed that the waves propagate over long distances so that the boundary effects can be neglected.

Applying the *principle of superposition of waves,*

$$s = s_1 + s_2$$

or $$s = a \cos 2\pi\nu_1 t + a \cos 2\pi\nu_2 t$$

or $$s = a(\cos 2\pi\nu_1 t + \cos 2\pi\nu_2 t)$$

or $$s = a\left(2\cos\frac{2\pi\nu_1 t + 2\pi\nu_2 t}{2}\cos\frac{2\pi\nu_1 t - 2\pi\nu_2 t}{2}\right)$$

or $$s = 2a \cos 2\pi \frac{(\nu_1 + \nu_2)}{2} t \cos 2\pi \frac{(\nu_1 - \nu_2)}{2} t$$

or $$s = \left[2a \cos 2\pi \left(\frac{\nu_1 - \nu_2}{2} \right) t \right] \cos 2\pi \left(\frac{\nu_1 + \nu_2}{2} \right) t$$

or $$s = \mathrm{A} \cos 2\pi \left(\frac{\nu_1 + \nu_2}{2} \right) t$$

where $\mathrm{A} \left[= 2a \cos 2\pi \left(\frac{\nu_1 - \nu_2}{2} \right) t \right]$ is the amplitude of the resultant wave. It may be noted that the frequency of the resultant wave is the average of the frequencies ν_1 and ν_2 of the superposing wave trains.

The amplitude A of the resultant wave is a function of time. A varies between + $2a$ and $-2a$. The amplitude A is *maximum, *i.e.,* + $2a$ or – $2a$ when

$$\cos \pi(\nu_1 - \nu_2)t = \pm 1$$

or $$\cos \pi(\nu_1 - \nu_2)t = \cos n\pi$$

where n = 0, 1, 2,.......

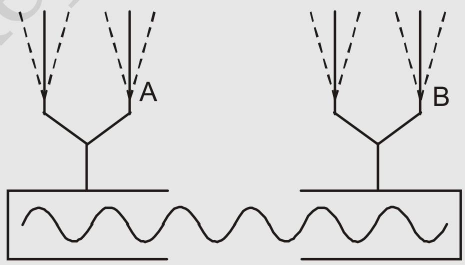
or $$\pi(\nu_1 - \nu_2)t = n\pi$$

or $$(\nu_1 - \nu_2)t = n$$

or $$t = \frac{n}{\nu_1 - \nu_2} = 0, \ \frac{1}{\nu_1 - \nu_2}, \ \frac{2}{\nu_1 - \nu_2}, \ \frac{3}{\nu_1 - \nu_2}, \ldots.$$

Experimental Demonstration of Beats

Two identical tuning forks are placed on two sound boxes as shown. Attach a little wax to one prong of tuning fork A. Set the two forks into vibration. Beats will be heard. On changing the amount of wax, the number of beats per second will change.



So, the time interval between two successive maxima is $\frac{1}{\nu_1 - \nu_2}$.

Similarly, the amplitude A is minimum (zero) when

$$\cos \pi(\nu_1 - \nu_2)t = 0 \quad \text{or} \quad \cos \pi(\nu_1 - \nu_2)t = \cos(n + \frac{1}{2})\pi$$

or $$(\nu_1 - \nu_2)t = (n + \frac{1}{2})$$

*Since amplitude is maximum $\therefore$ intensity is also maximum. It is proportional to $4a^2$.

or $$t = \frac{n + \frac{1}{2}}{\nu_1 - \nu_2} = \frac{\frac{1}{2}}{\nu_1 - \nu_2}, \frac{\frac{3}{2}}{\nu_1 - \nu_2}, \frac{\frac{2}{2}}{\nu_1 - \nu_2}, \ldots\ldots$$

So, the time interval between two successive minima is $\frac{1}{\nu_1 - \nu_2}$.

Thus, we find that maxima and minima occur at regular intervals of $\frac{1}{\nu_1 - \nu_2}$. So, the beat frequency is $(\nu_1 - \nu_2)$. This is equal to the difference in the frequencies of the two superposing wave trains.

Graphical representation of beats

Fig. 5.23 illustrates the phenomenon of beats for two harmonic waves of frequencies 11 Hz and 9 Hz. The amplitude of the resultant wave shows beats at a frequency of 2 Hz.

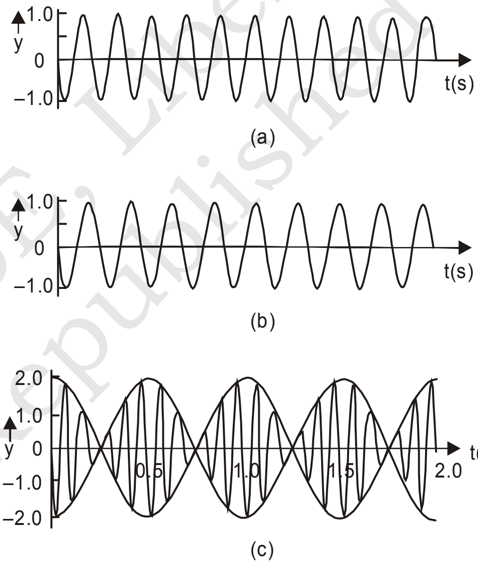


Fig. 5.23. Superposition of two harmonic waves, one of frequency 11 Hz. (*a*), and the other of frequency 9 Hz. (*b*), giving rise to beats of frequency 2 Hz, as shown in (*c*).

Uses of Beats

(*a*) To determine unknown frequency

The tuning fork of unknown frequency is sounded with a standard tuning fork of known frequency so that the beats are heard. The number of beats heard per second is determined. This is equal to the difference of 'unknown frequency' and 'known frequency'. Let N be the frequency of the standard tuning fork. Let '*a*' beats be heard per second. Then the unknown frequency is either (N + *a*) or (N – *a*).

To decide about the positive or negative sign, one of the prongs of the tuning fork of unknown frequency is **loaded with wax.** This decreases the frequency. Now, if the two tuning forks are sounded together, we will not hear '*a*' beats per second. If the number of beats heard per second is greater than *a*, then (N – *a*) was the correct frequency. If on loading, the number of beats heard per second is less than *a*, then (N + *a*) was the correct frequency of the fork.

If instead of loading one prong, it is **filed,** then the reverse results will be true.

Note that when a prong is filed a little, it becomes lighter and its frequency of vibration increases.

(*b*) Use in music. (*i*) *For tuning musical instruments.* The tension in the string of one of the two instruments is altered till beats are heard. This will occur at nearly equal frequencies. Keep on adjusting carefully till the beats disappear. Now, the two instruments are in tune. (*ii*) Sometimes in an orchestra, a deliberate 'beating' sound is produced. This gives the effect of a sonorous vibrating sound and is generally appreciated in musical performance.

(*c*) Use in electronics. Electronic beat frequency oscillators are commonly used to generate a beat frequency (BF) which is audible. Also in modern radio receivers, ultrasonic beats are generated and radio reception is obtained.

(*d*) Use in mines. The presence of dangerous gases in mines may be detected by the use of beats.

Example 15. *In an experiment, it was observed that a tuning fork and a sonometer wire gave 5 beats per second both when the length of wire was 1 m and 1.05 m. Calculate the frequency of the fork.*

Solution. Let the frequency of the fork be ν. At the smaller length of the sonometer wire (l_1 = 1 m), the frequency of the wire must be higher *i.e.*, $\nu_1 = \nu + 5$; and at the larger length (l_2 = 1.05 m), the frequency must be lower.

$\therefore \qquad \nu_2 = \nu - 5$

According to the law of length, $\dfrac{\nu_1}{\nu_2} = \dfrac{l_2}{l_1}$

$$\frac{\nu + 5}{\nu - 5} = \frac{1.05}{1.00}$$

On solving, we get ν = **205 Hz**

Example 16. *Two tuning forks A and B when sounded together give 4 beats/s. A is in unison with the note emitted by a 0.96 m length of a sonometer wire under a certain tension. B is in unison with 0.97 m length of the same wire under the same tension. Calculate the frequencies of the forks.*

Solution. A is in unison with a smaller length of the wire as compared to B. So, A has higher frequency as compared to B. Let ν be the frequency of A.

Then, $$\frac{\nu - 4}{\nu} = \frac{0.96}{0.97} = \frac{96}{97} \qquad \left[\because \ \nu \propto \frac{1}{l}\right]$$

or $$1 - \frac{4}{\nu} = 1 - \frac{1}{97} \quad \text{or} \quad \frac{4}{\nu} = \frac{1}{97}$$

or $$\nu = 4 \times 97 \text{ Hz} = \mathbf{388\ Hz}$$

5.25. DOPPLER EFFECT

The apparent change in the frequency of sound when the source of sound, the observer and the medium are in relative motion is called ***Doppler effect.***

Doppler effect applies to waves in general. This effect has been named after German-born Austrian Physicist Christian Johann Doppler (1803–1853).

Whenever there is relative motion between a listener (or observer) and a source of sound, the pitch or frequency of sound appears to be changed. If the source of sound is approaching the listener or the listener is approaching the source of sound or both are approaching each other, then the frequency of sound appears to be higher than the true frequency. If the source of sound is receding away from the listener or the listener is receding away from the source of sound or both are receding away from each other, then the frequency of sound appears to be lower than the true frequency.

Let us now derive expressions for the apparent frequency of sound in different cases. While deriving these expressions, we make the following **assumptions :**

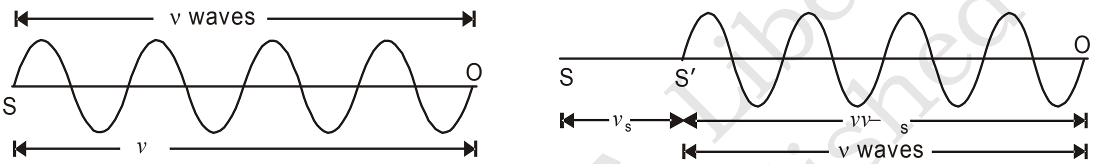
(*i*) The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.

(*ii*) The velocity of the source and the observer is less than the velocity of sound.

(*iii*) The velocity of sound is always positive.

Case I. Source in motion, Observer at rest, Medium at rest

Suppose the source S and the observer O are separated by distance v, where v is the velocity of sound. Let ν be the frequency of the sound emitted by the source. Then, ν waves *will be emitted by the source in 1 second.* These ν waves will be accommodated in distance v [Fig. 5.24 (*a*)]. Let the source start moving towards the observer with velocity v_s. After one second, the ν waves will be crowded in distance $(v - v_s)$ [Fig. 5.24 (*b*)]. Now, the observer shall feel that he is listening to sound of wavelength λ' and frequency ν'.



(a) Both source and observer at rest

(b) Source moving towards the observer

Fig. 5.24

Now, $$\nu' = \frac{v}{\lambda'} \quad \text{or} \quad \nu' = \frac{v}{v - v_s / \nu}$$

or $$\nu' = \frac{v\nu}{v - v_s} \quad \text{or} \quad \nu' = \frac{v}{v - v_s}\nu$$

So, as the source of sound approaches the observer, the apparent frequency ν' becomes greater than the true frequency ν.

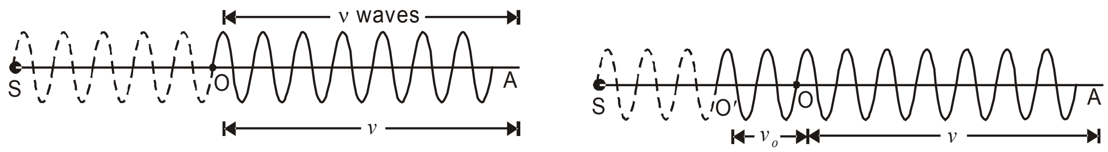
If the source is receding away from the observer, then the apparent frequency is given by

$$\nu' = \frac{v}{v + v_s}\nu$$

Case II. Observer in motion, Source at rest, Medium at rest

Let the source and observer occupy positions marked S and O respectively in Fig. 5.25 (*a*). Now take a point A such that OA = v. If both S and O are in their respective places, then ν waves given by S would be crossing O in 1 second and would fill the space OA (= v). In one second, O moves towards S with velocity v_o such that OO′ = v_o. So, the observer has received not only the ν waves occupying OA but has also received additional number of waves occupying the distance OO′. Thus in one second, the observer receives waves occupying the space AO′ such that

$$AO' = v + v_o$$



(a) Both source and observer are at rest (b) Observer moving towards source

Fig. 5.25

Number of waves in distance v = ν

Number of waves in unit distance = $\dfrac{\nu}{v}$

Number of waves in distance $(v + v_o)$ $= \dfrac{\nu}{v}(v + v_o)$

Apparent frequency, $\nu'' = \dfrac{\nu}{v}(v + v_o)$

$$\nu'' = \frac{v + v_o}{v}\nu$$

If the observer is moving away from the source, then the apparent frequency is given by

$$\nu'' = \frac{v - v_o}{v}\nu$$

Case III. When both the Source and Observer are moving towards each other

When the source moves towards a stationary observer,

$$\nu' = \frac{v}{v - v_s}\nu$$

Again, when the observer moves towards a stationary source,

$$\nu'' = \frac{v + v_o}{v}\nu$$

When both the source and observer move towards each other, then apparent frequency is given by

$$\nu''' = \frac{v + v_o}{v} \times \frac{v}{v - v_s}\nu \quad \text{or} \quad \nu''' = \frac{v + v_o}{v - v_s}\nu$$

If both the source and observer move in the direction of sound, then

$$\nu''' = \frac{v - v_o}{v - v_s}\nu$$

5.26. APPLICATIONS OF DOPPLER'S PRINCIPLE

(*i*) To determine the velocity of a star, galaxy etc.

Doppler's effect can be used to determine the *velocity of approach or recession of a heavenly body towards or away from the Earth.* When light from a star is examined by a spectroscope, the spectrum is found to consist of several well-defined spectral lines. If the star is approaching the Earth, a shift of spectral lines occurs towards the violet end of the spectrum. This indicates a decrease in wavelength.

When the star is receding away from the Earth, the spectral lines shift towards the red end of the spectrum indicating an increase in wavelength. These changes of wavelength on account of motion of star are called ***spectral shifts***. These help us to calculate the velocity of approach or the velocity of recession of the star.

Let the star be receding away from the Earth with velocity v. Then applying Doppler's effect, the apparent frequency of the light waves coming from the star is given by

$$\nu' = \frac{c}{c+v}\nu$$

where c is the velocity of light and ν is the true frequency of light waves.

$$\therefore \quad \frac{\nu'}{\nu} = \frac{c}{c+v}$$

But $$\nu' = \frac{c}{\lambda'} \quad \text{and} \quad \nu = \frac{c}{\lambda}$$

where λ' and λ are the apparent wavelength and true wavelength respectively.

$$\therefore \quad \frac{c}{\lambda'} \times \frac{\lambda}{c} = \frac{c}{c+v} \quad \text{or} \quad \frac{\lambda}{\lambda'} = \frac{c}{c+v}$$

or $$\frac{\lambda'}{\lambda} = \frac{c+v}{c} = 1 + \frac{v}{c}$$

or $$\frac{\lambda'}{\lambda} - 1 = \frac{v}{c} \quad \text{or} \quad \frac{\lambda' - \lambda}{\lambda} = \frac{v}{c}$$

or $$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \quad \text{or} \quad \Delta\lambda = \frac{v}{c}\lambda$$

By knowing the value of $\Delta\lambda$, we can calculate the velocity v of the star with respect to Earth. It has been generally observed that the wavelength of light received from the stars shifts slightly towards the red end of the spectrum. This '**red shift**' shows that the stars are receding away from us. So, our universe is expanding.

(*ii*) Radar

It measures not only the distance and location of an aeroplane but also its velocity by determining the frequency shift.

We know that $\nu' = \dfrac{c}{c - v_s}\nu = \nu\left(\dfrac{c - v_s}{c}\right)^{-1} = \nu\left(1 + \dfrac{v_s}{c}\right) = \nu + \nu\dfrac{v_s}{c}$

or $\nu' - \nu = \dfrac{v_s}{c}\nu$ or $\Delta\nu = \dfrac{v_s}{c}\nu$ or $v_s = c\dfrac{\Delta\nu}{\nu}$

So, by determining the frequency shift $\Delta\nu$, v_s can be calculated. This has to be halved to get the approach velocity of the aeroplane.

Example 17. *Determine the velocity of sound when the frequency appears to be double the actual frequency to a stationary observer.*

Solution. $\nu' = \dfrac{v}{v - v_s}\nu$

Now, $\nu' = 2\nu$ $\therefore$ $2\nu = \dfrac{v}{v - v_s}\nu$

or $2v - 2v_s = v$ or $v = 2v_s$ or $v_s = \dfrac{v}{2}$

The source should approach the stationary observer with a velocity equal to **half the velocity of sound**.

Example 18. *A factory siren whistles a note of frequency 680 Hz. A man travelling in a car at 108 km h^{-1} moving towards the factory hears the whistle. What is the apparent frequency of the sound as heard by him? Given : speed of sound in air = 340 m s^{-1}.*

Solution. $v_o = 108 \text{ km h}^{-1} = 108 \times \dfrac{5}{18} \text{ m s}^{-1} = 30 \text{ m s}^{-1}$

$v = 340 \text{ m s}^{-1}$, $\nu = 680$ Hz

$\nu' = \dfrac{v + v_o}{v}\nu = \dfrac{340 + 30}{340} \times 680 \text{ Hz} = \mathbf{740\ Hz}$

Example 19. *Two railway trains, each moving with a velocity of 108 km h^{-1}, cross each other. One of the trains gives a whistle whose* *frequency is 750 Hz. What will be the apparent frequency for passengers sitting in the other train before crossing ? Given : speed of sound = 330 m s*$^{-1}$.

Solution. $\upsilon_s = 108 \text{ km h}^{-1} = 108 \times \frac{5}{18} \text{ m s}^{-1} = 30 \text{ m s}^{-1}$

$$\upsilon_o = 108 \text{ km h}^{-1} = 30 \text{ m s}^{-1}$$

Note that the source and the observer are approaching.

$\therefore$ Apparent frequency, $\nu' = \dfrac{\upsilon + \upsilon_o}{\upsilon - \upsilon_s}\nu$

$\therefore$ $$\nu' = \frac{330+30}{330-30} \times 750 \text{ Hz} = \mathbf{900 \text{ Hz}}$$

5.27 RELATION BETWEEN LOUDNESS AND INTENSITY

Intensity of sound represents the sound energy that flows per second across a unit area held normal to the direction of flow.

This is an objective physical definition. The feeling in the listener's mind is spoken of as loudness. Thus, a sound of high intensity possesses a greater loudness.

(*i*) According to **Weber-Fechner law**, the loudness L of sound is directly proportional to the logarithm of intensity I.

$\therefore$ $$L \propto \log I \text{ or } L = K \log I$$

Here, K is a constant of proportionality.

(*ii*) Consider two sounds of same frequency having intensities I_1 and I_0 respectively. Let L_1 and L_0 be their corresponding loudness.

Then, $$L_1 = K \log_{10} I_1 \text{ and } L_0 = K \log_{10} I_0$$

Intensity level, $$L = L_1 - L_0 = K [\log_{10} I_1 - \log_{10} I_0]$$

or $$L = K \log_{10} \left[\frac{I_1}{I_0}\right]$$

(*iii*) Let I_0 represents the standard reference intensity (also called zero level of intensity). Its value is 10^{-12} W m^{-2}. It corresponds to the threshold audibility of a healthy human ear at a frequency of 1000 Hz.

If K = 1, then L is measured in bel. [The unit is named in honour of Alexander Graham Bell, the inventor of Telephone.]

Now, $$L = \log_{10}\left[\frac{I_1}{I_0}\right] \text{ bel}$$

If $$I_1 = 10I_0, \text{ then } L = \log_{10}\frac{10I_0}{I_0} = \log_{10}10 = 1 \text{ bel}$$

The intensity level of sound is said to be one bel if the intensity of sound is ten times the zero level of intensity.

The intensity level of sound will be 2 bel if the intensity of sound is 100 times the zero level of intensity.

(*iv*) Since bel is a large unit, therefore, a smaller unit called decibel (dB) is used.

$$1\text{dB} = \frac{1}{10}\text{ bel.}$$

Again, $$L = 10\log_{10}\left[\frac{I_1}{I_0}\right] \text{ decibel}$$

If $$L = 1 \text{ decibel, then } \log_{10}\left[\frac{I_1}{I_0}\right] = \frac{1}{10} = 0.1$$

or $$\frac{I_1}{I_0} = \text{antilog }(0.1) = 1.2589 \approx 1.26$$

We can conclude from here that a 26 percent increase in the intensity raises the intensity level by 1 decibel. It is interesting to note that it is the smallest change in intensity level that a healthy human ear can detect.

If $$I_1 = 100\, I_0,$$

then $$L = 10\log_{10}\left[\frac{100\, I_0}{I_0}\right] = 10\log_{10}100$$

$$= 10\log_{10}10^2 = 20\log_{10}10$$

$$= 20 \text{ decibels}$$

So, if the louder of the two sounds is 100 times more intense, then the two sounds differ by 20 decibels. Similarly, if the louder of the two sounds is 1000 times more intense, then the two sounds will differ by 30 decibels.

REVIEW EXERCISES

Do the review exercises in your notebook.

A. Multiple Choice Questions

1. A tuning fork of frequency 512 Hz makes 4 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was

(*a*) 510 Hz (*b*) 514 Hz

(*c*) 516 Hz (*d*) 508 Hz.

2. A transverse wave is represented by $y = \text{A} \sin (\omega t - kx)$.

For what value of the wavelength is the wave velocity equal to the maximum particle velocity?

(*a*) $\frac{\pi \text{A}}{2}$ (*b*) πA

(*c*) 2πA (*d*) A.

3. Two strings A and B are slightly out-tune and produce beats of frequency 5 Hz. Increasing the tension in B reduces the beat frequency to 3 Hz. If the frequency of string A is 450 Hz, calculate the frequency of string B.

(*a*) 460 Hz (*b*) 455 Hz

(*c*) 445 Hz (*d*) 440 Hz.

4. A resonance pipe is open at both ends and 30 cm of its length is in resonance with an external frequency 1.1 kHz. If the speed of sound is 330 m s^{-1} which harmonic is in resonance ?

(*a*) first (*b*) second

(*c*) third (*d*) fourth.

5. When two progressive waves $y_1 = 4 \sin (2x - 6t)$ and $y_2 = 3 \sin \left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5.

6. A wave motion is described by $y(x, t) = a \sin(kx - \omega t)$. Then the ratio of the maximum particle velocity to the wave velocity is
   (a) ωa (b) $\frac{1}{ka}$
   (c) $\frac{\omega}{k}$ (d) ka.
7. Velocity of sound in air is 320 m s^{-1}. A pipe closed at one end has a length of 1 m. Neglecting end correction, the air column in the pipe cannot resonate with sound of frequency
   (a) 80 Hz (b) 240 Hz
   (c) 320 Hz (d) 400 Hz
8. A whistle is blown from the tower of a factory with a frequency of 220 Hz. The apparent frequency of sound heard by a worker moving towards the factory with a velocity of 30 m s^{-1} is (Velocity of sound = 330 m s^{-1})
   (a) 280 Hz (b) 200 Hz
   (c) 300 Hz (d) 240 Hz
9. The frequencies of two tuning forks A and B are respectively 1.5% more and 2.5% less than that of the tuning fork C. When A and B are sounded together, 12 beats are produced in 1 second. The frequency of the tuning fork C is
   (a) 200 Hz (b) 240 Hz
   (c) 360 Hz (d) 300 Hz
10. Two pipes are each 50 cm in length. One of them is closed at one end while the other is open at both ends. The speed of sound in air is 340 m s^{-1}. The frequency at which both the pipes can resonate is
   (a) 680 Hz (b) 510 Hz
   (c) 85 Hz (d) none of the above.

B. Fill in the Blanks

1. A train moving towards a hill at a speed of 72 km h^{-1} sounds a whistle of frequency 500 Hz. A wind is blowing from the hill at a speed of 36 km h^{-1}. If the speed of sound in air is 340 m s^{-1}, the frequency heard by a man on the hill is__________ .
2. When two sound sources of the same amplitude but of slightly different frequencies n_1 and n_2 are sounded simultaneously, the sound one hears has a frequency equal to __________ .
3. A travelling wave represented by $y = A \sin(\omega t - kx)$ is superimposed on another wave represented by $y = A \sin(\omega t + kx)$. The resultant is __________ .

4. Two identical piano wires, kept under the same tension T have a fundamental frequency of 600 Hz. The fractional increase in the tension of one of the wires which will lead to occurrence of 6 beats/s when both the wires oscillate together would be __________.

5. Sound waves travel at 350 m s^{-1} through warm air and at 3500 m s^{-1} through brass. The wavelength of a 700 Hz acoustic wave as it enters brass from warm air __________ .

6. Tube A has both ends open while tube B has one end closed. Otherwise they are identical. Their fundamental frequencies are in the ratio __________ .

7. The speed of sound in a gas of density ρ at a pressure P is proportional to __________ .

8. The intensity ratio of two waves at a point is $\frac{4}{9}$. The amplitude ratio will be __________ .

9. Two sound waves travel in the same direction in a medium. The amplitude of each wave is A and the phase difference between the two waves is 120°. The resultant amplitude will be __________ .

10. A plane progressive wave is given by

$$y = 2 \cos 6.284\ (330t - x).$$

The period of the wave is __________ .

C. Very Short Answer Questions

1. What is the range of frequency of audible sound?
2. Why does sound travel faster in iron than in air?
3. What kind of waves help the bats to find their way in the dark?
4. The velocity of sound in air is 332 m s^{-1}. Find the frequency of the fundamental note of an open pipe 50 cm long.
5. In which gas, hydrogen or oxygen, will sound have greater velocity?
6. In a resonance tube, the second resonance does not occur exactly at three times the length at first resonance. Why?
7. The frequency of the fundamental note of a tube closed at one end is 200 Hz. What will be the frequency of the fundamental note of a similar tube of the same length but open at both ends?
8. A wave transmits energy. Can it transmit momentum?
9. A string has a linear density of 0.25 kg m^{-1} and is stretched with a tension of 25 N. What is the velocity of the wave?
10. By how much the wave velocity increases for 1°C rise of temperature?

D. Short Answer Questions

1. A tuning fork of unknown frequency gives 4 beats with a tuning fork of frequency 310 Hz. It gives the same number of beats on filing. Find the unknown frequency.
2. The string of a violin emits a note of 540 Hz at its correct tension. The string is bit taut and produces 4 beats per second with a tuning fork of frequency 540 Hz. Find the frequency of the note emitted by this taut string.
3. The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. Find the length of the air column. [End correction may be neglected]
4. In the following series of resonant frequencies, one frequency (lower than 400 Hz) is missing : 150, 225, 300, 375 Hz (a) What is the missing frequency? (b) What is the frequency of the seventh harmonic?
5. Flash and thunder are produced simultaneously. But thunder is heard a few second after the flash is seen. Why?

E. Long Answer Questions

1. Densities of oxygen and nitrogen are in the ratio 16 : 14. At what temperature the speed of sound in oxygen will be the same as at 15°C in nitrogen?
2. Calculate the speed of sound in oxygen from the following data. The mass of 22.4 litre of oxygen at STP (T = 273 K and P = 1.0×10^5 N m^{-2}) is 32 g, the molar heat capacity of oxygen at constant volume is $C_v = 2.5$ R and that at constant pressure is $C_p = 3.5$ R.
3. A sound wave of frequency 400 Hz is travelling in air at a speed of 320 m s^{-1}. Calculate the difference in phase between two points on the wave 0.2 m apart in the direction of travel.
4. A displacement wave is represented by

   $$y = 0.25 \times 10^{-3} \sin (500t - 0.025\, x),$$

   where y, t and x are in cm, second and metre respectively. Deduce (*i*) the amplitude (*ii*) the period (*iii*) the angular frequency (*iv*) the wavelength. Deduce also the amplitude of particle velocity and particle acceleration.
5. Two harmonic waves have the same displacement amplitude of 4×10^{-5} cm and their angular frequencies are 500 rad s^{-1} and 5000 rad s^{-1}. Calculate (*i*) particle velocity amplitude, and (*ii*) particle acceleration amplitude.

SEMESTER-2 (Period-VI)

TOPIC 6

Light

P11CH6

6.1. LIGHT

Light *is that form of energy* (*optical energy*), *which helps us in seeing objects from which it comes or from which it is reflected e.g.,* sun gives us light and hence we can see the sun.

Light *may also be defined as that form of energy which produces in us the sensation of vision* (*sight*).

(We do **not see light** because light itself is not visible, since **no energy is visible**.)

Light falling on the objects, returns from them and then falls on our eyes, which makes objects visible. **Our eye is a natural optical instrument.**

6.2. SOURCES OF LIGHT

Objects from which the light comes out are called ***sources of light***. Some sources are **natural** while many others are **man–made**. For us on the earth, **Sun is the most important natural source of light.** Electric lamps, oil lamps and candles are some of the man made sources of light.

Objects which are visible through the light emitted by them, are called ***luminous sources.*** Sun, stars, bulbs, candles, etc. are luminous objects.

There are certain objects which do not emit their own light but still we can see them *e.g.,* table, chair, etc. This is due to the reason that when light from some source is incident on these objects, then it gets reflected or scattered. This reflected or scattered light enters our eyes and we are able to see these objects. *These objects, which do not emit* *their own light but become visible due to the light reflected or scattered by them are called* ***non-luminous objects.*** Hence, we can say that **light is the form of energy which produces in us the sensation of vision.**

6.3. NATURE OF LIGHT

Depending on the type of observation and level of understanding, there are two theories about the nature of light *i.e.*, **wave theory of light and particle theory of light**.

(*i*) **Wave theory of light.** According to this theory, light travels from the source in the form of a wave. The waves are found to be **transverse electromagnetic waves**. *These waves do not require any material medium for their* **propagation**.

The **speed of these waves is 3×10^8 m s^{-1}** in vacuum and slightly less in air. The speed of light is represented by the symbol *c*. Its actual value is, $c = 299{,}792{,}458$ m s^{-1}.

The wavelength of visible light ranges from 4×10^{-7} m to 8×10^{-7} m and is very small as compared to the size of usual objects. *Light waves travel (propagate) from one point (source) to other in a straight line, called the* ***ray of light***. The rays are taken to be perpendicular to **wave front** (front of the wave).

(*ii*) **Particle theory of light.** According to this theory, *light is made up of some elementry particles, called* **photons** *which travel in straight line with very high speed.*

Photons have **only energy** and **no rest mass** and **no charge.**

This particle nature has been used to explain a new additional phenomenon, called photoelectric effect.

6.4. PHOTON ENERGY AND COLOUR OF LIGHT

White light consists of **seven** colours namely **V**iolet, **I**ndigo, **B**lue, **G**reen, **Y**ellow, **O**range and **R**ed (remembered by the word **VIBGYOR**).

Photon of **red light has minimum frequency** (minimum energy, $E = h\nu$) and the **red light has maximum wavelength.**

Photon of violet **light has maximum frequency** (maximum energy, $E = h\nu$) and the **violet light has minimum wavelength**.

Thus we can say that, red light photons have minimum energy *i.e.*, **red light is least energetic** and violet light photons have maximum energy *i.e.*, **violet light is most energetic.**

6.5. DUAL NATURE OF LIGHT

It has been found that some phenomena like diffraction, interference and polarisation of light can be explained only if light is considered to be of wave nature whereas some other phenomena like reflection of light, refraction of light, photoelectric effect cannot be explained by wave nature of light but can be explained only if light is considered to be made up of particles.

Hence we can say that *light has a dual nature, particle nature as well as wave nature.*

According to the particle nature, light consists of photons having frequency ν (nu) and energy, $E = h\nu$ (where h is Planck's constant).

According to the wave nature, light consists of waves having wavelength λ (lamda) and velocity, $c = \nu\lambda$.

Combining both relations, we get, energy,

$$E = h\nu = \frac{hc}{\lambda} \quad i.e., \; E \propto \frac{1}{\lambda}$$

6.6. RAY AND BEAM OF LIGHT

1. **Ray of Light: A *ray of light is a straight line along which light travels*.** In Fig. 6.1, OP represents a ray of light.

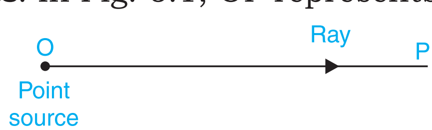


Fig. 6.1. Ray of light.

2. **Beam of light:** A bundle of rays associated with a point source, form a beam of light.

   It is shown in Fig. 6.2.

If the rays of a beam come out of the point source (O), then beam is **diverging** [Fig. 6.2(*a*)].

If the rays of a beam meet at a point (O), then beam is **converging** [Fig. 6.2(*b*)].

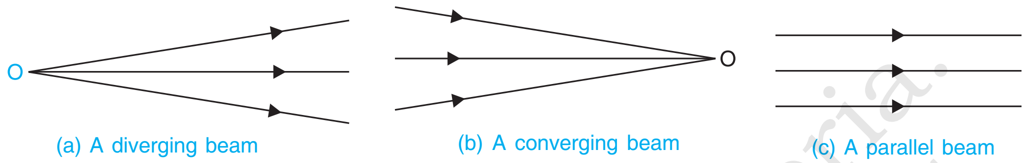


Fig. 6.2. Beam of light.

A beam of light, in which all the rays are parallel to each other, is called **parallel beam** of light.

6.7. OPTICAL MEDIUM

Substance, surrounding a source of light through which light travels, is called ***optical medium*** *or* ***simply medium.*** A medium can be : transparent, opaque or translucent.

(*i*) **Transparent medium:** Medium through which light can completely pass, is called a transparent medium.
Examples: Air, water, glass.

(*ii*) **Opaque medium:** Medium through which no light can pass, is called an opaque medium.
Examples: Wood, wall, metals.

(*iii*) **Translucent medium:** Medium through which light passes only partially, is called a translucent medium.
Examples: Tracing paper, oil-soaked paper.

6.8. PROPAGATION OF LIGHT

Light travels in a straight line from a source as long as it remains in one medium and is not obstructed by any object. This mode of propagation of light is called **rectilinear propagation**. Activity 6.1 demonstrates that light travels in a straight line.

ACTIVITY 6.1

To Demonstrate that Light Travels in a Straight Line

Materials Required

Three rectangular pieces of cardboard, a candle, and a lighter

Procedure

1. Take three rectangular pieces of cardboard.
2. Make holes in each of them in the centre such that all the holes are at exactly the same horizontal level.
3. Make the cardboard stand straight and parallel on a table using wooden supports.
4. Make sure that the holes in all the three cardboard pieces are aligned.
5. Light the candle and keep it on the table with its flame at the level of the hole in the first cardboard.
6. Now keep your eye in front of the third cardboard and adjust the cardboards such that you can see the candle flame through the holes [Fig. 6.3 (a)].

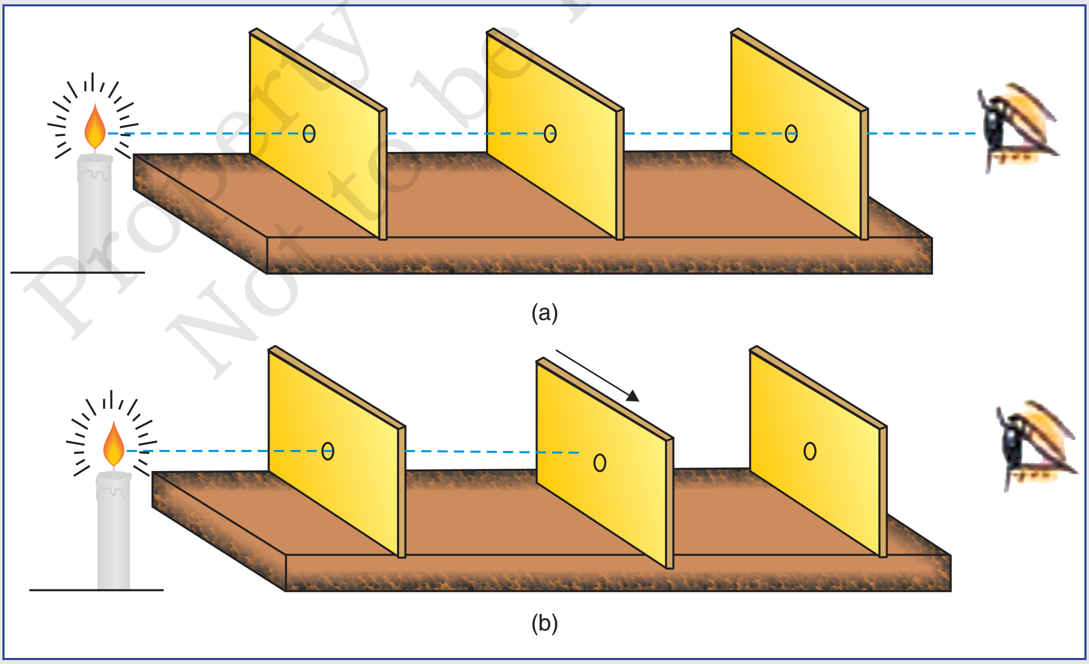


Fig. 6.3. Experiment to show that light travels in a straight line

7. Move one of the cardboards slightly to misalign its hole to the others and observe.
8. What do you observe [Fig. 6.3 (b)]?

Observation

You will observe that the flame can only be seen when the holes are exactly in a straight line. If you disturb one of the cardboards, you will no longer be able to see the flame. This activity clearly proves that light travels in a straight line.

6.9. REFLECTION AND REFRACTION OF LIGHT

Reflection

You can see an object only when light falls on it. When light falls on a surface, it bounces off the surface and strike our eyes. It makes us see the things. *The bouncing back of light rays from a surface is called* ***reflection***.

Refraction

Refraction *is the bending of light as it crosses the interface between two different transparent media.*

6.10. TYPES OF REFLECTION

There are two types of reflection, known as Regular and Diffused reflection.

ACTIVITY 6.2

Imagine that parallel rays are incident on an irregular surface as shown in Fig. 6.4. The laws of reflection are valid at each point of the surface. Use these laws to construct reflected rays at various points. Are they parallel to one another? You will find that these rays are reflected in different directions (Fig. 6.5).

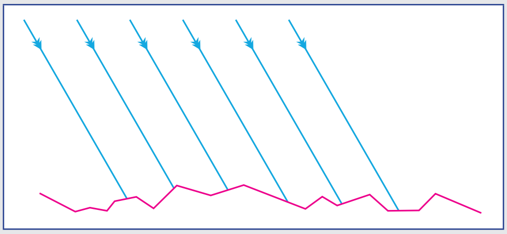


Fig. 6.4. Parallel rays incident on an irregular surface

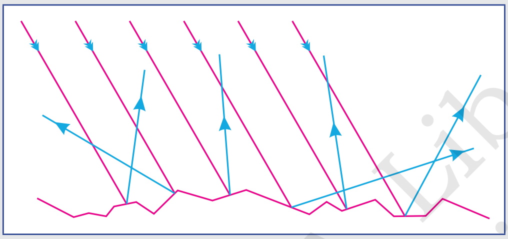


Fig. 6.5. Rays reflected from irregular surface

When all the parallel rays reflected from a plane surface are not parallel, the reflection is known as **diffused** or **irregular** reflection. Remember that the diffused reflection is not due to the failure of the laws of reflection. It is caused by the irregularities in the reflecting surface, like that of a cardboard.

On the other hand reflection from a smooth surface like that of a mirror is called **regular reflection** (Fig. 6.6). Images are formed by regular reflection.

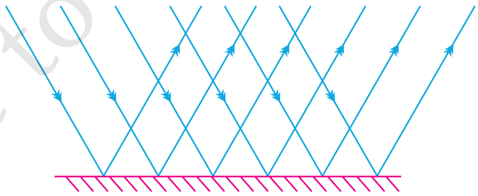


Fig. 6.6. Regular reflection

6.11. FORMATION OF SHADOWS

We now know that light travels in a straight line. So, an opaque object blocks the light falling on it. This creates an area of darkness on the side of the object away from light. This area of darkness is called the **shadow** of the object.

The following three things are required for a shadow to form (Fig. 6.7):

- a source of light;
- an opaque object; and
- a screen or surface behind the object.

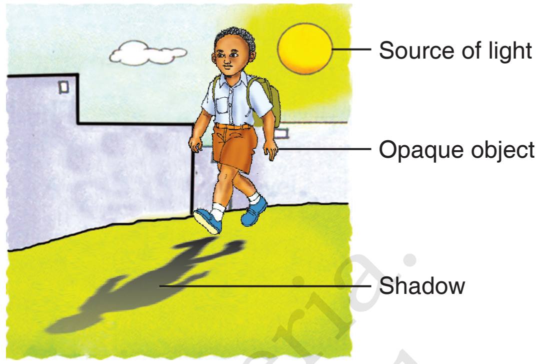


Fig. 6.7. Formation of shadow

A shadow cannot form if any of these is absent. This explains why we cannot see a shadow in the dark. It is only when light rays are obstructed by an opaque object that we get a shadow of the object. Activity 6.3 will make us understand the formation of shadow and its characteristics.

ACTIVITY 6.3

To Obtain a Shadow and Study its Characteristics

Materials Required

A torch, a few small opaque objects of different shapes and sizes, and a white screen (a piece of cardboard covered with white paper).

Procedure

1. Turn on the torch and place any opaque object in front of it.
2. Hold the screen on the other side of the object to get the shadow.
3. Ask your friend to trace out the outline of the shadow on the screen.
4. Now, keeping the positions of the torch and the screen intact, move the object closer to the torch. What do you see?
5. Note the size of the shadow.
6. Repeat steps 1 to 5 for different objects.
7. Does the colour of the shadow change with size or for various different objects?

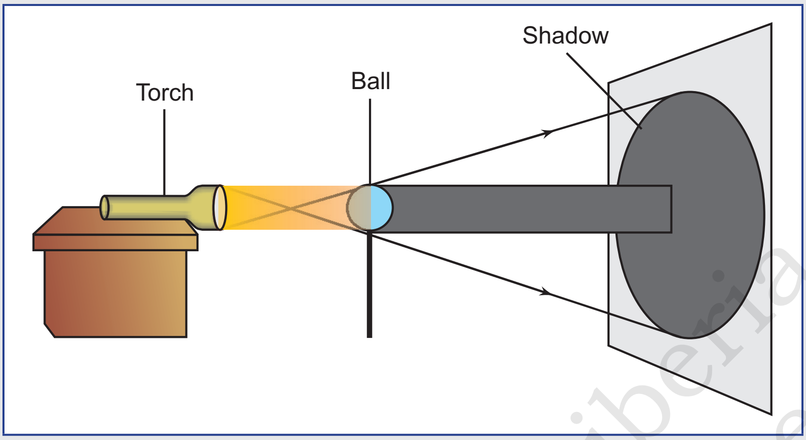


Fig. 6.8. Demonstration of formation of shadow

Observation

The shadow becomes bigger when the object is moved closer to the torch, and smaller when it is moved closer to the screen. The colour of the shadow is always black.

Characteristics of a Shadow

A shadow has the following three characteristics:

1. It is always black, regardless of the colour of the object used to make the shadow.
2. It only shows the shape or outline of the object and not the details.
3. The size of a shadow varies. It depends on the distance between the object and the source of light, and the distance between the object and the screen.

Identification of Umbra and Penumbra

If the light source is a point, then all objects will have one kind of shadow behind them. But if the light source is a sphere, then every object has behind it a core shadow known as **umbra** and a sort of side-way shadow known as **penumbra**. Similarly, the distant light source forms penumbra and umbra irrespective of its shape.

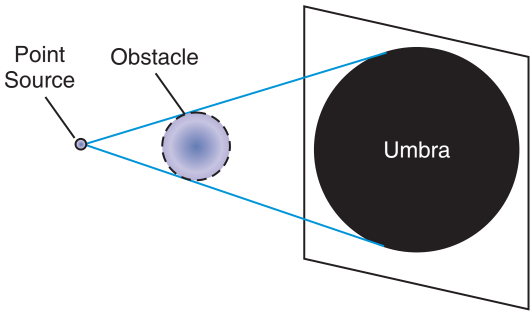


Fig. 6.9. Formation of umbra

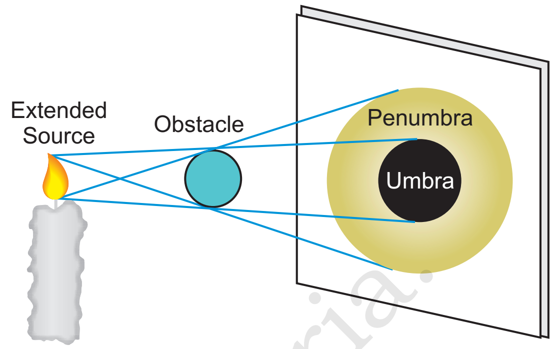


Fig. 6.10. Formation of umbra and penumbra

6.12. FORMATION OF ECLIPSES

Eclipse is the blocking of light from the sun by the interference of the moon or earth. There are two types of eclipse.

(*a*) Solar eclipse

(*b*) Lunar eclipse

Solar Eclipse

Solar eclipse is the eclipse of the sun. It occurs when the moon passes between the sun and the earth. The shadow of the moon may completely block the sun. This is called **total solar eclipse.** And when only a portion of the sun is out of view, it is called **partial solar eclipse**.

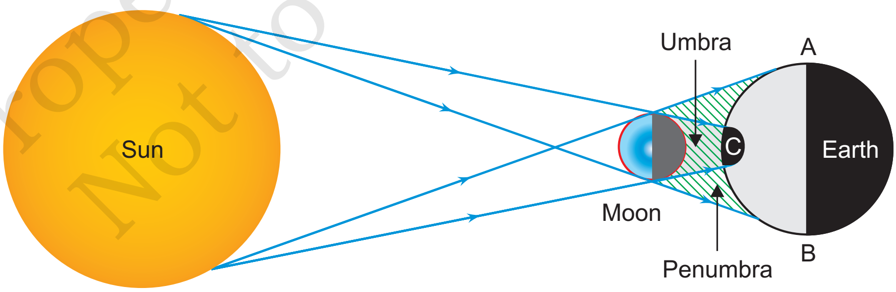


Fig. 6.11. Solar eclipse.

Lunar Eclipse

Lunar eclipse (eclipse of the moon) occurs when the earth passes between the sun and the moon. The shadow of the earth falls on the moon, blocking its view, partially or totally.

Umbra: It is the darkest part of the shadow. Here all the light from the source is blocked.

Penumbra: It is the region where the shadow is partial.

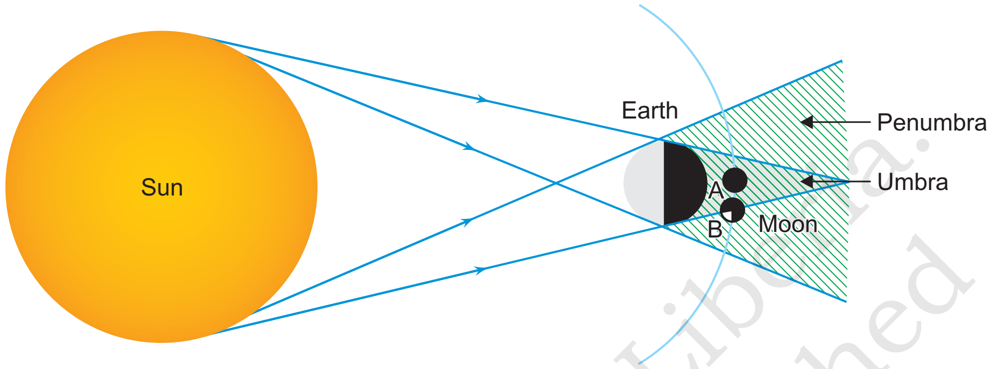


Fig. 6.12. Lunar eclipse

When the whole sheet of paper is spread on the table, it represents one plane. The incident ray, the normal at the point of incidence and the reflected ray are all in this plane. When you bend the paper you create a plane different from the plane in which the incident ray and the normal lie. Then you do not see the reflected ray. What does it indicate? It indicates that **the incident ray, the normal at the point of incidence and the reflected ray all lie in the same plane**. This is another **law of reflection**.

Thus, the law of reflection states that

(*i*) the angle of incidence is always equal to the angle of reflection.

(*ii*) the incident ray, the normal at the point of incidence and the reflected ray all lie in the same plane.

6.13. PINHOLE CAMERA IMAGE FORMATION AND MAGNIFICATION

A pinhole camera consists of a light proof box with a pinhole on one end and a screen of tracing paper at the other end. It has no lens. The image is formed by light travelling in straight line from an object to the screen.

A common use of the pinhole camera is to capture the movement of the sun over a long period of time. It is popular for observing solar eclipses.

Operation of the Pinhole Camera

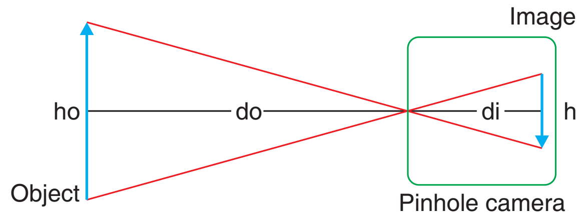


Fig. 6.13

Aim the camera at a bright object in a darkened room. You will see an upside down image on the tracing paper. The upside down image is formed because light rays travel in a straight line. The light rays from the top of the object travel through the pinhole and strike the bottom of the screen of pinhole camera. The light rays from the bottom of the object travel through pinhole and strike the top of the screen of the pinhole camera, thus forming the upside down image.

If a line is drawn through the pinhole and perpendicular to both the image and the object, it can be shown by similar triangles that:

$$\text{Magnification } m = \frac{\text{height of image}}{\text{height of object}} = \frac{\text{distance of image}}{\text{distance of object}}$$

$$m = \frac{hi}{ho} = \frac{di}{do}$$

Example 1. *What is the height of an image if an object 8.0 cm high is 125 cm from a pinhole camera that is 21 cm long?*

Solution.

$$\frac{hi}{ho} = \frac{di}{do}$$

$$hi = \frac{diho}{do}$$

$$hi = \frac{21 \cancel{\text{cm}} \times 8.0 \text{ cm}}{125 \cancel{\text{cm}}}$$

$$\boxed{hi = 1.3 \text{ cm}}$$

What is the magnification?

$$m = \frac{di}{do}$$

$$m = \frac{21 \cancel{\text{cm}}}{125 \cancel{\text{cm}}}$$

$$\boxed{m = 0.17}$$

Example 2. *What is the actual size of an object if the magnification is 0.20 and the image is 3.5 cm high?*

Solution.

$$m = \frac{hi}{ho}$$

$$0.20 = \frac{3.5\text{ cm}}{ho}$$

$$ho = \frac{3.5}{0.20}\text{ cm} = 17.5\text{ cm}$$

6.14. REFLECTION OF LIGHT

When light is incident on the surface of an object then it may be reflected, absorbed or transmitted.

When whole of the light, incident on the surface of an object, is absorbed by the object then object appears black. Our hairs appear black because they absorb most of the light incident on them.

If the object allows the incident light to pass through it then object is said to be transparent *e.g.,* sheet of ordinary glass. *When light passes through a transparent medium, it bends from its path and this phenomenon is called* ***refraction of light.***

If light falls on an opaque polished smooth surface (medium), then it returns back in the same medium.

For example, a polished silver mirror reflects back most of the light incident on it.

This phenomenon of sending back of rays of light in the same medium when they are incident on a smooth polished surface is called ***reflection of light.***

Some objects reflect more light and some objects reflect less light. Objects with polished shining surfaces reflect more light than the object having dull surfaces. **Silver metal is the best reflector of light.**

6.15. REFLECTION OF LIGHT FROM A PLANE POLISHED SURFACE

Since silver metal is the best reflector of light, hence ordinary mirrors are made by depositing a very thin layer of silver on one side of the plane glass sheet. **Then thin silver layer is coated with red paint, so as to protect the silver coating.** Reflection of light from a plane mirror takes place at the interface of glass and silver.

In our future discussion and while making diagram, a plane mirror is represented by a straight line with a number of oblique lines on its back side.

In Fig. 6.14, XY is section of a plane polished surface mirror. A ray of light PO strikes the surface at O and is returned back along OQ. An arrow on PO and OQ gives us the direction of propagation of ray of light.

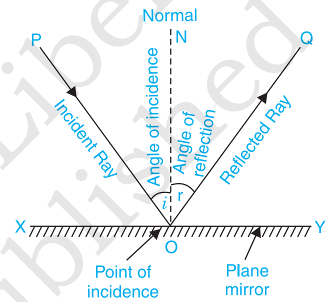


Fig. 6.14. Reflection of light from a plane polished surface.

*The returning back of the light in same medium is called **reflection.***

Some important terms associated with the reflection from a plane polished surface are given below:

(*i*) **Reflecting surface:** The surface from which the light is reflected, is called the reflecting surface. In diagram, *XY* is the reflecting surface, (**Actually** XY is the section of a reflecting surface, made by the plane of the book page which is perpendicular to it). Silver metal is one of the best reflectors of light.

(*ii*) **Point of incidence:** The point on the reflecting surface at which a ray of light strikes, is called the point of incidence. In Fig. 6.14, O is the point of incidence.

(*iii*) **Normal:** Normal is a line, perpendicular to the reflecting surface, at the point of incidence. In Fig. 6.14, NO is the normal.

(*iv*) **Incident ray:** The ray of light which strikes the reflecting surface at the point of incidence is called the incident ray. In Fig. 6.14, PO is the incident ray.

(*v*) **Reflected ray:** The ray of light which is sent back by the reflecting surface from the point of incidence, is called the reflected ray. In Fig. 6.14, OQ is the reflected ray.

(*vi*) **Angle of incidence:** The angle which the incident ray makes with the normal at the point of incidence is called the angle of incidence.

It is represented by the symbol $\angle i$. In Fig. 6.14, angle PON is the angle of incidence.

(*vii*) **Angle of reflection:** The angle which the reflected ray makes with the normal at the point of incidence is called the angle of reflection.

It is represented by the symbol $\angle r$. In Fig. 6.14, angle QON is the angle of reflection.

(*viii*) **Plane of incidence:** The plane in which the normal and the incident ray lie, is called the plane of incidence. In Fig. 6.14, the plane of the book-page, is the plane of incidence.

(*ix*) **Plane of reflection:** The plane in which the normal and the reflected ray lie, is called the plane of reflection. In Fig. 6.14, the plane of the book-page, is the plane of reflection.

6.16. MIRRORS

The polished surfaces used in the study of reflection of light, are called mirrors.

These are of two types: (*i*) *Plane mirrors* and (*ii*) *Spherical mirrors.*

(*i*) **Plane mirrors:** If the polished reflecting surface is plane, the mirror is called a plane mirror. Figure 6.15 shows, *XY* as the section of a plane mirror.

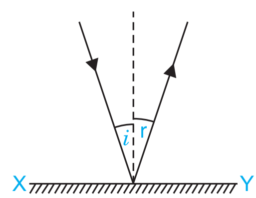


Fig. 6.15. Plane mirror.

(*ii*) **Spherical mirrors:** Spherical mirror is a part of a hollow sphere whose one side is polished.

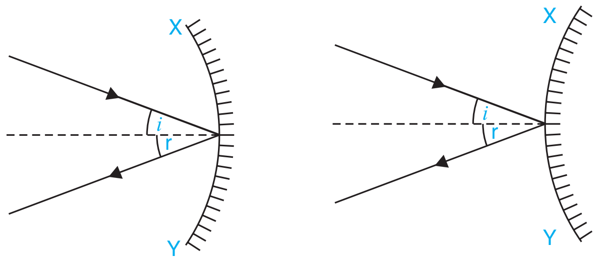
Spherical mirrors are of *two* types:

(*a*) *Concave mirror.* It is polished on the convex side and reflection from this mirror takes place from the concave side.

It is shown in Fig. 6.16 (*a*).

(*b*) *Convex mirror.* It is polished on the concave side and reflection from this mirror takes place from the convex side.

It is shown in Fig. 6.16 (*b*).



(a) Concave mirror (b) Convex mirror

Fig. 6.16. Spherical mirrors.

6.17. LAWS OF REFLECTION OF LIGHT

When light is incident on a smooth surface (mirror) then it gets reflected in accordance with the **two laws of reflection**. These laws of reflection are given below.

First law: *The incident ray, the reflected ray and the normal at the point of incidence, all lie in the same plane*. In Fig. 6.14, incident ray *PO*, reflected ray *OQ* and the normal *ON*, all lie in the same plane *i.e.*, plane of the paper.

Second law: The angle of incidence is always equal to the angle of reflection. If angle of incidence is $\angle i$ and angle of reflection is $\angle r$ then,

$$\angle i = \angle r$$

When a ray of light falls normally on a mirror *i.e.*, at right angle, then angle of incidence, $(\angle i) = 0°$. Since angle of incidence is 0°, hence in accordance with the second law of reflection, angle of reflection will also be zero *i.e.*, $\angle r = 0°$.

In other words, we can say that *when a ray of light falls normally on a mirror, then reflected ray will also travel perpendicular the mirror i.e., when a ray of light falls on a mirror normally, it gets reflected back along the same path.*

Laws of reflection given above are equally applicable to all types of mirrors.

6.18. REFLECTION FROM SPHERICAL MIRRORS

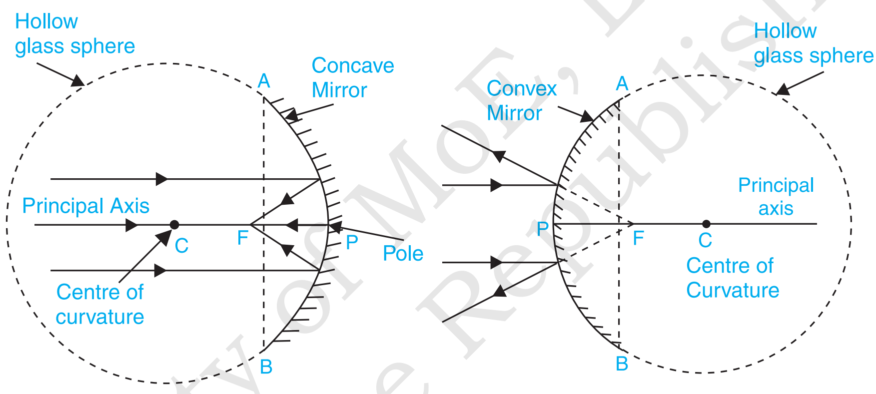
Some terms associated with spherical mirrors are given below:

(*i*) **Aperture:** *The diameter of the circular rim of the mirror is called the*

aperture of the mirror. *Size of the mirror is usually referred to as aperture*. In Fig. 6.17, *AB* is the aperture of the mirror.

(*ii*) **Pole:** *The centre of the spherical mirror is called* ***pole of the mirror***. It lies on the surface of the mirror. It is the lowest point in case of a concave mirror and highest point in case of a convex mirror. ***All distances are measured from the pole of the mirror***. In Fig. 6.17, *P* is the pole of the mirror.

(*iii*) **Centre of curvature:** *Centre of curvature of a spherical mirror is the centre of the hollow sphere of which mirror is a part*. It lies outside the surface of the mirror. Every point on the surface of the spherical mirror lies at the same distance from it. In Fig. 6.17, *C* is the centre of curvature of the mirror.



(a) Concave mirror (b) Convex mirror

Fig. 6.17. Spherical mirrors.

(*iv*) **Principal axis:** *The straight line passing through the pole of the mirror and the centre of curvature of the mirror, is called principal axis of the mirror.*

(*v*) **Principal focus:** *It is a point on the principal axis of the mirror, such that the rays incident on the mirror, parallel to the principal axis, after reflection actually meet at this point (in case of a concave mirror) or appear to come from this point (in case of a convex mirror).* In Fig. 6.17, *F* is the principal focus of the mirror.

(*vi*) **Radius of curvature:** *The distance between the pole and the centre of curvature of the mirror is called the radius of curvature of the mirror*. It is equal to the radius of the hollow sphere of which, the mirror is a part. In Fig. 6.17, *PC* is the radius of curvature of the mirror. It is represented by the symbol *R*.

(*vii*) **Focal length:** *The distance between the pole and principal focus of the mirror is called the focal length of the mirror.* In Fig. 6.17, *PF* is the focal length of the mirror. It is represented by the symbol ***f***. For a **concave mirror focal length is negative**.

(*viii*) **Principal section:** *A section of the spherical mirror cut by a plane passing through its centre of curvature and the pole of the mirror, is called the **principal section of the mirror***. It contains the principal axis. In diagram, *APB* is the principal section of the mirror cut by the plane of the book page.

6.19 ELECTROMAGNETIC SPECTRUM (including elementary facts about their uses)

Electromagnetic waves cover a wide range of frequencies or wavelengths. The classification of electromagnetic waves does not have sharp boundaries. This is because the classification of electromagnetic waves is done according to their main source and different sources may produce waves in overlapping ranges of frequencies.

Electromagnetic spectrum *is the orderly distribution of electromagnetic radiations in accordance with their wavelength or frequency.* The usual classification of the electromagnetic spectrum is summarised below:

1. Radio frequency waves. (*a*) These have wavelengths ranging from a few kilometre down to 0.3 m. The frequency range is from a few Hz to 10^9 Hz.

(*b*) Radio waves reach us from **extraterrestrial sources.** The Sun is a major source of radio waves. These often interfere with radio and TV reception on Earth. Jupiter is also an active source of radio emissions.

(*c*) Mapping the radio transmissions from extraterrestrial sources, known as **radio astronomy,** has provided information about the universe that is often not obtainable using optical telescopes. Since the Earth's atmosphere does not absorb strongly at radio wavelengths, radio astronomy provides certain advantages over optical, infrared or microwave astronomy on Earth.

(*d*) ***Properties of Radiowaves.*** **(*i*)** They are electromagnetic waves. **(*ii*)** They travel with a velocity of 3×10^8 m s^{-1} in vacuum. **(*iii*)** They can be reflected, refracted and diffracted.

(*e*) *Uses of Radiowaves.* **(*i*)** The early uses were maritime, for sending telegraphic messages using Morse code between ships and land. **(*ii*)** They are used in AM broadcast radio and FM broadcast radio. **(*iii*)** They are used in Aviation voice radios and Marine voice radios. **(*iv*)** Civil and military voice services use short wave radio to contact ships at sea, aircraft and isolated settlements. **(*v*)** Radar detects things at a distance by bouncing radio waves off them. **(*vi*)** They are used in radio remote controls. **(*vii*)** They are used in radio and TV communication systems. **(*viii*)** They are used in radio-astronomy. **(*ix*)** Cellular phones use radio waves to transmit voice communication in the UHF band.

2. Microwaves. (short-wavelength radio waves) **(*a*)** The wavelengths of microwaves range from 0.3 m down to 10^{-3} m. The frequency range is from 10^9 Hz upto 3×10^{11} Hz.The microwave region is also designated as UHF (ultra-high frequency relative to radio frequency).

(*b*) *Properties of Microwaves.* **(*i*)** They are electromagnetic waves. **(*ii*)** They travel with a velocity of 3×10^8 m s^{-1} in vacuum. **(*iii*)** They can be reflected, refracted and diffracted. **(*iv*)** When absorbed by matter, they produce heat. **(*v*)** Microwaves pass easily through the earth's atmosphere with less interference than longer wavelengths. **(*vi*)** There is much more bandwidth in the microwave spectrum than in the rest of the radio spectrum. **(*vii*)** They can be used to transmit power over long distances.

(*c*) *Uses of Microwaves.* **(*i*)** They are used in the analysis of very fine details of atomic and molecular structure. **(*ii*)** They are used for cooking. Microwave ovens are an interesting domestic application of these waves. In such ovens, the frequency of the microwaves is selected to match the resonant frequency (3 GHz) of water molecules so that energy from the waves is transferred efficiently to the kinetic energy of the molecules. This raises the temperature of any food containing water. **(*iii*)** They are used in communication satellite transmissions. **(*iv*)** Due to their short wavelengths, they are suitable for the radar systems used in aircraft navigation. In fact, radar uses microwave radiation to detect the range, speed and other characteristics of remote objects. **(*v*)** Cable TV, Internet and cellphone networks make use of lower microwave frequencies.

3. Infrared rays. **(*a*)** The infrared spectrum covers wavelengths from 10^{-3} m down to 7.8×10^{-7} m (or 7800 Å). The frequency range is from 3×10^{11} Hz up to 4×10^{14} Hz.

(*b*) Infrared rays are produced by hot bodies and molecules. Broadly speaking, following are the laboratory sources for the **production** of infrared rays. The underlying principle is excitation of atoms and molecules. This may involve vibration and bending of molecules.

(*i*) *Laser.* It produces highly monochromatic infrared rays. CO_2 Laser gives infrared rays of wavelength 10.6 μm. He-Ne Laser gives infrared rays of wavelengths 0.69 μm, 1.19 μm and 3.39 μm.

(*ii*) *Filament of Nernst Lamp.* It is made from a mixture of zirconium, thorium and cesium. When current flows through such a filament, it gets heated. At a temperature of nearly 1200 K, infrared rays are emitted.

(*c*) *Earth as an infrared emitter.* The surface of earth absorbs visible radiation from the sun and re-emits a major portion of this energy as infrared back into the atmosphere.

(*d*) For **detection** of infrared rays, we use bolometers, thermopiles, photo conducting cells etc.

(*e*) Properties of infrared rays. (*i*) They are electromagnetic waves. **(*ii*)** They travel with a velocity of $3 \times 10^8 \text{ m s}^{-1}$ in vacuum. **(*iii*)** They show interference effects. **(*iv*)** They can be polarised. **(*v*)** They affect photographic plate. **(*vi*)** They show heating effect. **(*vii*)** Smoke is more transparent to infrared than to visible light. **(*viii*)** Under fog conditions, infrared can travel through long distances because of their low scattering.

(*f*) Uses of infrared rays. (*i*) They are used in night vision devices during warfare. This is because they can pass through haze, fog and mist. **(*ii*)** Infrared rays are used to take photographs in darkness. **(*iii*)** They are used to keep the green houses warm. **(*iv*)** They are used in revealing the secret writings on the ancient walls. **(*v*)** They are used in muscular therapy *i.e.,* to treat muscular strains. IR bulbs are used in muscular therapy. **(*vi*)** The infrared rays from the sun keep the earth warm. **(*vii*)** They provide electrical energy to a satellite by using solar cells. **(*viii*)** They are used in solar water heaters and cookers. **(*ix*)** They are used for producing dehydrated fruits. **(*x*)** They are used in weather forecasting through infrared photography.

4. Light or visible spectrum. This is a narrow band formed by the wavelengths to which our retina is sensitive. It extends from a wavelength of 7.8×10^{-7} m down to 3.8×10^{-7} m and frequencies from 4×10^{14} Hz up to 8×10^{14} Hz. Light is produced by atoms and molecules as a result of internal adjustment in the motion of their components, principally that of the electrons.

Colour	λ (in metre)	ν (in Hz)
Violet	$3.90 \times 10^{-7} - 4.55 \times 10^{-7}$	$7.69 \times 10^{14} - 6.59 \times 10^{14}$
Blue	$4.55 \times 10^{-7} - 4.92 \times 10^{-7}$	$6.59 \times 10^{14} - 6.10 \times 10^{14}$
Green	$4.92 \times 10^{-7} - 5.77 \times 10^{-7}$	$6.10 \times 10^{14} - 5.20 \times 10^{14}$
Yellow	$5.77 \times 10^{-7} - 5.97 \times 10^{-7}$	$5.20 \times 10^{14} - 5.03 \times 10^{14}$
Orange	$5.97 \times 10^{-7} - 6.22 \times 10^{-7}$	$5.03 \times 10^{14} - 4.82 \times 10^{14}$
Red	$6.22 \times 10^{-7} - 7.80 \times 10^{-7}$	$4.82 \times 10^{14} - 3.84 \times 10^{14}$

The sensitivity of the eye also depends on the wavelength of light. This sensitivity is maximum for wavelengths of approximately 5.6×10^{-7} m. Because of the relation between colour and wavelength or frequency, an electromagnetic wave of well-defined wavelength or frequency is also called a monochromatic wave (monos-one ; chromos colour).

5. Ultraviolet Rays. (*a*) These rays were discovered by Ritter in 1801.

Ultraviolet rays are electromagnetic waves whose wavelength ranges from 6×10^{-10} m (0.6 nm) to 4×10^{-7} m (400 nm). The frequency ranges from 8×10^{14} Hz to 5×10^{17} Hz.

Their energy is of the order of magnitude of the energy involved in many chemical reactions. This accounts for many of their chemical effects.

(b) Ultraviolet rays are a part of the solar spectrum. These waves are produced by atoms and molecules in electrical discharges. They can be produced by passing discharge through hydrogen and xenon. They can also be produced by the arcs of mercury and iron.

The sun is a very powerful source of ultraviolet radiation. This fact is mainly responsible for suntans. Exposure to UV radiation induces the production of more melanin, causing tanning of the skin. UV radiation is absorbed by ordinary glass. Hence, one cannot get tan or sunburn through glass window.

Welders wear special glass goggles or face masks with glass windows to protect their eyes from large amount of UV produced by welding arcs.

(*c*) *Properties of ultraviolet rays.* **(*i*)** They are electromagnetic waves. **(*ii*)** They travel with a velocity of 3×10^8 m s^{-1} in vacuum. **(*iii*)** They obey the laws of reflection and refraction. **(*iv*)** They show interference and polarisation. **(*v*)** They affect photographic plate. **(*vi*)** They show photoelectric effect. **(*vii*)** They cannot pass through glass. However, they can pass through quartz, fluorite and rock salt. **(*viii*)** They can cause fluorescence in certain materials. **(*ix*)** When skin is exposed to sunlight, ultraviolet rays synthesise vitamin D. **(*x*)** These rays are very harmful to the living tissues. **(*xi*)** Brief exposure to ultraviolet radiation causes common sunburn, but long-term exposure can lead to more serious effects, including skin cancer.

(*d*) *Uses of ultraviolet rays.* **(*i*)** They are used to preserve food stuffs as the rays kill germs. **(*ii*)** They are used to make drinking water free from bacteria. UV lamps are used to kill germs in water-purifiers. **(*iii*)** Ultraviolet absorption spectra are used in the study of molecular structure and the arrangement of electrons in the external shells of atoms. **(*iv*)** Ultraviolet rays have medical applications. Since these rays destroy bacteria therefore they are used for sterilising surgical instruments. **(*v*)** They are used in detecting the invisible writings, forged documents, counterfeit currency notes and finger prints in forensic laboratory. **(*vi*)** Ultraviolet rays are used for checking the mineral samples by making use of the fact that ultraviolet rays cause fluorescence.

6. X-rays. This part of the electromagnetic spectrum extends from wavelengths of nearly 10^{-9} m down to wavelengths of nearly 6×10^{-12} m or frequencies between 3×10^{17} Hz and 5×10^{19} Hz. X-rays were discovered in 1895 by the German physicist W. Roentgen when he was studying cathode rays. **X-rays are produced** by the inner or more tightly bound electrons in atoms. Another source of X-rays is the *bremsstrahlung* or decelerating radiation.

They are **used** in medical diagnosis because the relatively greater absorption of X-rays by bone as compared with tissue allows for a fairly well-defined pattern on a photographic film. They also, as a result of the chemical processes they induce, cause serious damage to living tissues and organisms. It is for this reason that X-rays are used for treatment of cancer, to destroy diseased tissue. It should be emphasised that even a small amount of X-rays also destroys some good tissue and exposure to a large dose of X-rays may cause enough destruction to produce sickness or death.

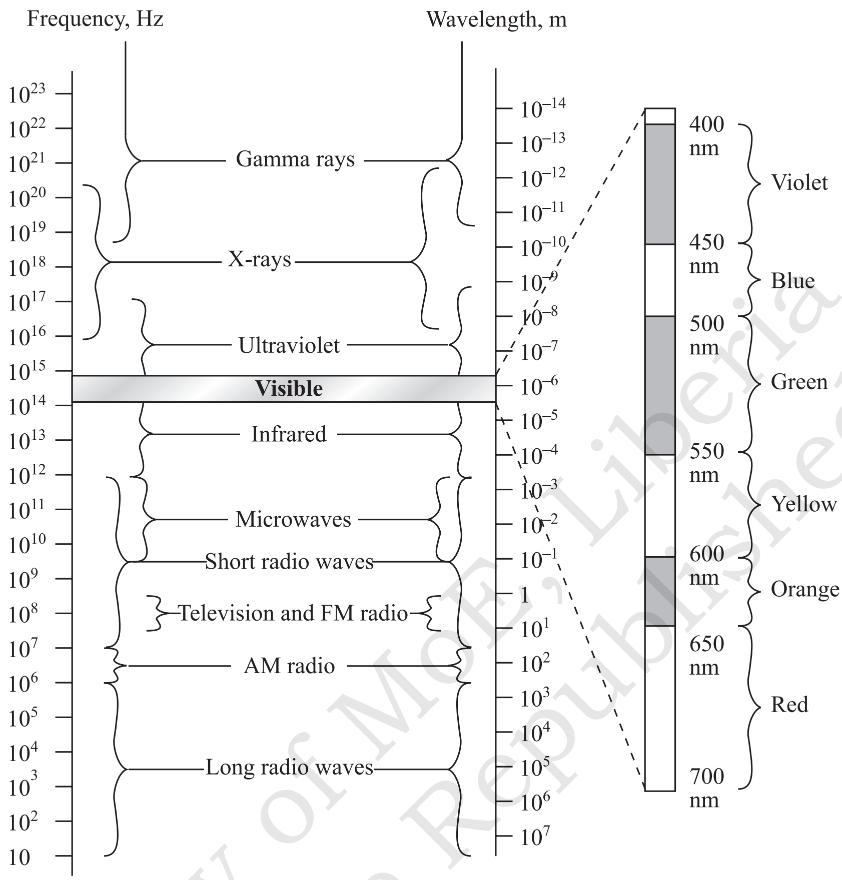


Fig. 6.18. The electromagnetic spectrum, with common names for various part of it. The various regions do not have sharply defined boundaries.

7. Gamma rays. These electromagnetic waves are of nuclear origin. They overlap the upper limit of the X-ray spectrum. Their wavelength ranges from nearly 10^{-10} m to well below 10^{-14} m, with a corresponding frequency range from 3×10^{18} Hz to more than 3×10^{22} Hz. The energies of these waves are of the same order of magnitude as those involved in nuclear processes and therefore the absorption of γ-rays may produce some nuclear changes. **Gamma rays are produced** by many radioactive substances and are present in large quantities in nuclear reactors.

Uses of Gamma Rays : Gamma rays are used:

(*i*) in radiotherapy for the treatment of malignant tumours.

(*ii*) to initiate some nuclear reactions.

(*iii*) to preserve food stuffs for a long time. This is because soft γ-rays can kill micro-organisms.

(*iv*) to study the structure of atomic nuclei.

REVIEW EXERCISES

Do the review exercises in your notebook.

A. Multiple Choice Questions

1. The energy whose presence makes the surrounding objects visible is:

(a) heat (b) sound
(c) light (d) electrical.

2. Medium through which light is fully passed, is called

(a) transparent (b) opaque
(c) transluscent (d) alloy.

3. Medium through which light cannot pass, is called

(a) transparent (b) opaque
(c) transluscent (d) alloy.

4. Medium through which light is partially passed, is called

(a) transparent (b) opaque
(c) transluscent (d) opaque transparent.

5. Angle of reflection is the angle between

(a) incident ray and normal to the surface
(b) incident ray and surface of the mirror
(c) reflected ray and surface of mirror
(d) reflected ray and normal to the surface.

6. In case of reflection from a spherical mirror, the image formed is

(a) always real
(b) always virtual
(c) real as well as virtual
(d) neither real nor virtual.

7. In sign convention to be followed, the mirror is kept with its reflecting face towards

(a) left (b) right
(c) upward (d) downward.

8. Image of the face has an enlarged size when seen in a mirror from a close distance. The mirror is

(a) plane (b) concave
(c) convex (d) parabolic

9. Bending of a ray of light, when it enters obliquely from one medim to other is called
   (a) reflection (b) refraction
   (c) dispersion (d) interference
10. The relation, $\frac{\sin i}{\sin r} = n$, is called
   (a) Snell's law (b) Newton's law
   (c) Joule's law (d) Boyle's law

B. Fill in the Blanks

1. __________ nature of light is used in our everyday life.
2. Light passes partially through __________ medium.
3. When two converging rays become incident on a convex mirror, the image formed is __________ .
4. Height of an inverted real image has a __________ sign.
5. For a convex mirror, magnification m is __________ one.
6. In refraction, a ray of light __________ when it enters obliquely in some other medium.
7. Image distance for the image on the right of the lens is __________ .
8. A lens is put over a printed page, if diminished image of the print is seen, then lens is __________ .
9. A ray of light passing through the optical centre of a lens goes __________.
10. If ${}^{a}n_{g} = 3/2$, then ${}^{g}n_{a}$ = __________ .

C. Very Short Answer Questions

1. Which is a converging mirror: a convex or a concave?
2. Can a magnified image be formed by a convex mirror?
3. What determines the focal length of a spherical mirror?
4. In a concave mirror, is the reflecting surface away from the centre of the sphere of which the mirror forms a part?
5. In a concave mirror, when is the size of image exactly equal to the size of the object?
6. What is the angle of incidence when a ray falls normally on a mirror?
7. Do the laws of reflection hold good in case of spherical mirrors?
8. What is meant by refraction of light?
9. What do you mean by an optical medium?
10. What is the approximate wavelength of X-rays?

D. Short Answer Questions

1. What is a mirror formula? Is it same for a convex and concave mirrors?
2. What is the focal length of a plane mirror?
3. A ray of light falls on a plane mirror making an angle of 60° with the mirror. Find the angle through which the ray gets deviated after reflection from the mirror.
4. An object is held at 30 cm in front of a convex mirror of focal length 15 cm. At what distance from the convex mirror should a plane mirror be held so that images in the two mirrors coincide with each other?
5. What is mirror formula? Does it change with the nature of the image formed? Express the mirror formula in terms of radius of curvature of the mirror.

E. Long Answer Questions

1. An erect image three times the size of the object is obtained with a concave mirror of radius of curvature 0.36 m. Find the position of the object.
2. The image formed by a convex mirror of radius of curvature 40 cm is a quarter of the object. Calculate the distance of the object from the mirror.
3. When an object is placed at a distance of 60 cm from a convex mirror, the magnification produced is 1/2. Where should the object be kept to get a magnification of $\frac{1}{3}$.
4. A concave lens of focal length 25 cm and a convex lens of focal length 20 cm are placed in contact with eachother. What is the power of this combination? Also, calculate focal length of the combination.
5. A concave lens has focal length of 15 cm. At what distance should an object from the lens be placed so that it forms an image at 10 cm from the lens? Also, find the magnification of the lens.